Chapter 10: Games and Strategic Behavior

Monday, July 19

WHAT IS A 'GAME'?

***basic elements of a game*:**

the players,

the strategies available to each player,

and the payoffs each player receives for each possible combination of strategies

HOW TO READ A PAYOFF MATRIX

		player B	
		B plays “left”	B plays “right”
player A	A plays “up”	10 5	3 1
	A plays “down”	3 4	12 2

If A plays up, and B plays left, then A gets 10, and B gets 5.

By convention, the payoff for player on the left (in this case, A) is listed first in each cell.

HOW TO READ A PAYOFF MATRIX

		B: left	B: right
A	up	10 5	3 1
A	down	3 4	12 2

If A plays up, and B plays left, then A gets 10, and B gets 5.

If A plays up, and B plays right, then A gets 3, and B gets 1.

If A plays down, and B plays left, then A gets 3, and B gets 4.

If A plays down, and B plays right, then A gets 12, and B gets 2.

QUESTION 1 (reading a payoff matrix)

		B: left	B: right
A	up	7, 3	5, 2
A	down	9, 6	10, 8

If A plays down, and B plays left, then what is A's payoff?

A) 6
B) 9
C) 7
D) 5
E) 10

answer to question 1

		B: left	B: right
A	up	7, 3	5, 2
A	down	9, 6	10, 8

If A plays down, and B plays left, then what is A's payoff?

A) 6
B) 9
C) 7
D) 5
E) 10

QUESTION 2 (reading a payoff matrix)

		B: left		B: right	
A	up	3	10	10	6
A	down	7	2	9	7

If A plays up, and B plays right, then what is B's payoff?

A) 10
B) 2
C) 7
D) 6
E) 3

answer to question 2

		B: left	B: right
A	up	3, 10	10, 6
	down	7, 2	9, 7

If A plays up, and B plays right, then what is B's payoff?

A) 10
B) 2
C) 7
D) 6
E) 3

CHICKEN

Biff

		swerve		straight	
Alex	swerve	6	6	5	8
	straight	8	5	0	0

Alex and Biff drive towards each other. At the last second, they must each simultaneously decide whether to swerve, or to continue driving straight towards the other guy. The worst outcome for both players is if they both go straight. The best outcome for either player is if he's the only one who goes straight.

ROCK PAPER SCISSORS

		Barney		
		rock	paper	scissors
Fred	rock	0 0	-1 +1	+1 -1
	paper	+1 -1	0 0	-1 +1
	scissors	-1 +1	+1 -1	0 0

Fred and Barney are playing rock-paper-scissors for money. If Fred plays paper, and Barney plays rock, then paper covers rock, so Barney has to give $1 to Fred.

DOMINANT STRATEGIES

B

A		left		right	
	up	4	4	1	6
	down	6	2	2	7

No matter what B does, A is better off if he plays down.

No matter what A does, B is better off if he plays right.

Hence, "down, right" is a *dominant strategy equilibrium*.

QUESTION 3 (dominant strategies)

A \ B	left	right
up	5, 9	5, 2
down	7, 1	9, 2

Does A have a dominant strategy?

A) Yes, his dominant strategy is to play up

B) Yes, his dominant strategy is to play down

C) No, he doesn't have a dominant strategy

answer to question 3

		B		
A		left		right	
	up	5	9	5	2
	down	7	1	9	2

Does A have a dominant strategy?

A) Yes, his dominant strategy is to play up

B) Yes, his dominant strategy is to play down

C) No, he doesn’t have a dominant strategy

QUESTION 4 (dominant strategies)

A \ B	left	right
up	5, 7	2, 3
down	6, 5	4, 2

Is there a dominant strategy equilibrium, i.e. a combination of strategies where both players are playing a dominant strategy?

A) Yes, it's up, left
B) Yes, it's up, right
C) Yes, it's down, left
D) Yes, it's down, right
E) No

answer to question 4

		B	
		left	right
A	up	5, 7	2, 3
	down	6, 5	4, 2

Is there a dominant strategy equilibrium, i.e. a combination of strategies where both players are playing a dominant strategy?

A) Yes, it's up, left
B) Yes, it's up, right
C) Yes, it's down, left
D) Yes, it's down, right
E) No

PRISONER’S DILEMMA

Jim and Mike are partners in crime. They’ve just committed a major crime, and the police are onto them, but they can’t prove it was them. However, the police do have conclusive evidence that Jim and Mike have committed a smaller crime, and they plan to use this as leverage.

The police put Jim and Mike in separate interrogation rooms so that they can’t talk to each other, and they give each of them the same ultimatum…

PRISONER'S DILEMMA

		Mike: confess	Mike: deny
Jim	confess	-10 -10	0 -20
	deny	-20 0	-1 -1

If neither criminal confesses, then the police can only convict them for the lesser crime, so that they each serve 1 year in prison.

If both confess, then they will be convicted of the major crime, and each serve 10 years in prison.

However, if only one criminal confesses, then the police will let him go free, and give the other criminal the maximum sentence of 20 years.

QUESTION 5 (prisoner's dilemma)

		Mike: confess	Mike: deny
Jim	confess	-10 -10	0 -20
	deny	-20 0	-1 -1

Is there a dominant strategy equilibrium?

A) Yes, it's where they both confess.
B) Yes, it's where they both deny.
C) Yes, it's either outcome where only one confesses.
D) No, there is no dominant strategy equilibrium.

answer to question 5

		Mike: confess	Mike: deny
Jim	confess	-10 -10	0 -20
	deny	-20 0	-1 -1

Is there a dominant strategy equilibrium?

A) Yes, it's where they both confess.

B) Yes, it's where they both deny.

C) Yes, it's either outcome where only one confesses.

D) No, there is no dominant strategy equilibrium.

NASH EQUILIBRIUM

In a dominant strategy equilibrium, then each player has a strategy that is best no matter what the other player is doing. However, this is not always the case.

		Biff			
		swerve		straight	
Alex	swerve	6	6	5	8
	straight	8	5	0	0

If Biff swerves, then Alex's best strategy is to go straight, but if Biff goes straight, then Alex's best strategy is to swerve.

NASH EQUILIBRIUM: CHICKEN

***Nash equilibrium*: any combination of strategies in which each player's strategy is his or her best choice, given the other players' choices**

		Biff: swerve	Biff: straight
Alex	swerve	6 6	5 8
	straight	8 5	0 0

In this example, there are two Nash equilibria, one where Alex is the chicken, and one where Biff is the chicken.

Note that if a combination of strategies is a dominant strategy equilibrium, then it's necessarily also a Nash equilibrium.

NASH EQUILIBRIUM: GENERIC EXAMPLE

		B: left		B: right	
A	up	5	7	2	6
A	down	6	2	3	3

		B: left		B: right	
A	up	5	7	2	6
A	down	6	2	3	3

QUESTION 6 (battle of the sexes)

		Kim: wrestling		Kim: opera	
Daniel	wrestling	20	5	0	0
Daniel	opera	0	0	5	20

Which is true?

A) There is no Nash equilibrium

B) The only Nash equilibrium is when they both go to the wrestling match

C) They only Nash equilibrium is when Daniel goes to the wrestling match, and Kim goes to the opera

D) There are 2 Nash equilibria: one in which they both go to the wrestling match, and one in which they both go to the opera

answer to question 6

		Kim: wrestling	Kim: opera
Daniel	wrestling	20, 5	0, 0
	opera	0, 0	5, 20

Which is true?

A) There is no Nash equilibrium

B) The only Nash equilibrium is when they both go to the wrestling match

C) They only Nash equilibrium is when Daniel goes to the wrestling match, and Kim goes to the opera

D) There are 2 Nash equilibria: one in which they both go to the wrestling match, and one in which they both go to the opera

QUESTION 6 (Nash equilibrium)

A \ B	left	right
up	8, 4	8, 2
down	3, 2	4, 6

Which is true?

A) up, left is the only Nash equilibrium

B) up, right is the only Nash equilibrium

C) down, right is the only Nash equilibrium

D) up, left and up, right are both Nash equilibria

E) up, left and down, right are both Nash equilibria

answer to question 6

		B	
		left	right
A	up	8 4	8 2
	down	3 2	4 6

Which is true?

A) up, left is the only Nash equilibrium

B) up, right is the only Nash equilibrium

C) down, right is the only Nash equilibrium

D) up, left and up, right are both Nash equilibria

E) up, left and down, right are both Nash equilibria

ZERO-SUM GAMES

		B: left	B: right
A	up	-3 3	3 -3
A	down	1 -1	2 -2

		B: left	B: right
A	up	2 8	8 2
A	down	6 4	7 3

Both of these games are zero-sum games.

Mathematically, this means that the sum of the players' payoffs are the same in every possible outcome. (In the first game, payoffs always sum to zero; in the second, payoffs always sum to 10.)

Intuitively, this means that there are no potential gains from cooperation; zero-sum games are 'strictly competitive'.

ZERO-SUM VS. NON-ZERO-SUM

zero-sum

A \ B	left	right
up	6 3	4 5
down	1 8	2 7

non-zero-sum

A \ B	left	right
up	4 7	1 5
down	2 9	6 8

In the game on the left, the sum of the players' payoffs is 9, no matter what. It is a zero-sum game.

In the game on the right, the sum of the players' payoffs is variable. It is a non-zero-sum game.

QUESTION 7 (zero sum games)

Which of the following payoff matrices represents a zero-sum (or constant-sum) game?

(A)

	left	right
up	7 8	5 1
down	6 0	9 6

(B)

	left	right
up	5 3	5 5
down	5 8	5 7

(C)

	left	right
up	9 2	3 8
down	4 7	6 5

(D)

	left	right
up	-3 -3	0 -9
down	-9 0	-1 -1

answer to question 7

Below are the sums of the players' payoffs for each outcome. Only in answer choice C to they always sum up to the same number. Thus, any gain by one player must come at the expense of a precisely equal loss by the other player.

(A)

	left	right
up	15	6
down	6	15

(B)

	left	right
up	8	10
down	13	12

(C)

	left	right
up	11	11
down	11	11

(D)

	left	right
up	-6	-9
down	-9	-2