

# **Chapter 15: Public Goods and Tax Policy**

**Monday, July 26**

# RIVALNESS AND EXCLUDABILITY

rival?

yes

no

yes

**private good**

**natural  
monopoly**  
(e.g. cable  
television)

excludable?

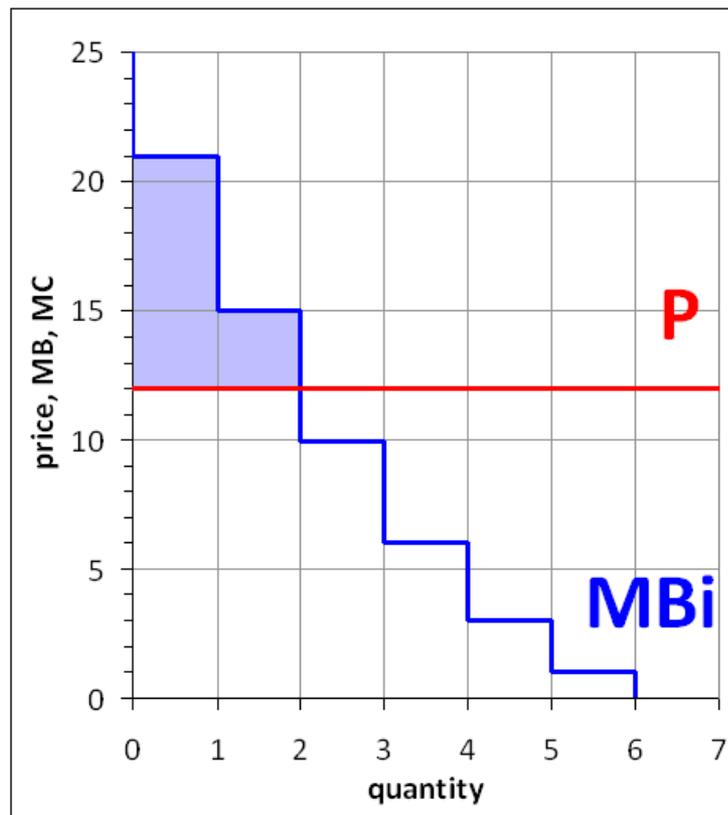
no

**common  
resource**  
(e.g. fish in the  
ocean)

**public good**  
(e.g. national  
defense)

# INDIVIDUAL BENEFIT AND DEMAND

Q	TB <sub>i</sub>	MB <sub>i</sub>
1	21	21
2	36	15
3	46	10
4	52	6
5	55	3
6	56	1
7	56	0



$$\begin{aligned}
 CS &= TB - TC \\
 &= 36 - 2 \times 12 \\
 &= 12
 \end{aligned}$$

or

$$\begin{aligned}
 CS &= \sum (MB - MC) \\
 &= (21 - 12) + (15 - 12) \\
 &= 12
 \end{aligned}$$

**Suppose that I live in a house with 4 other people, and we're deciding how many paintings to buy for the common room wall. My individual benefit from different numbers of paintings are as given above. If no one else bought any at all, and the price of a painting was \$12, then I'd buy 2 paintings and get a consumer surplus of 12, as shown above.**

## SOCIAL BENEFIT

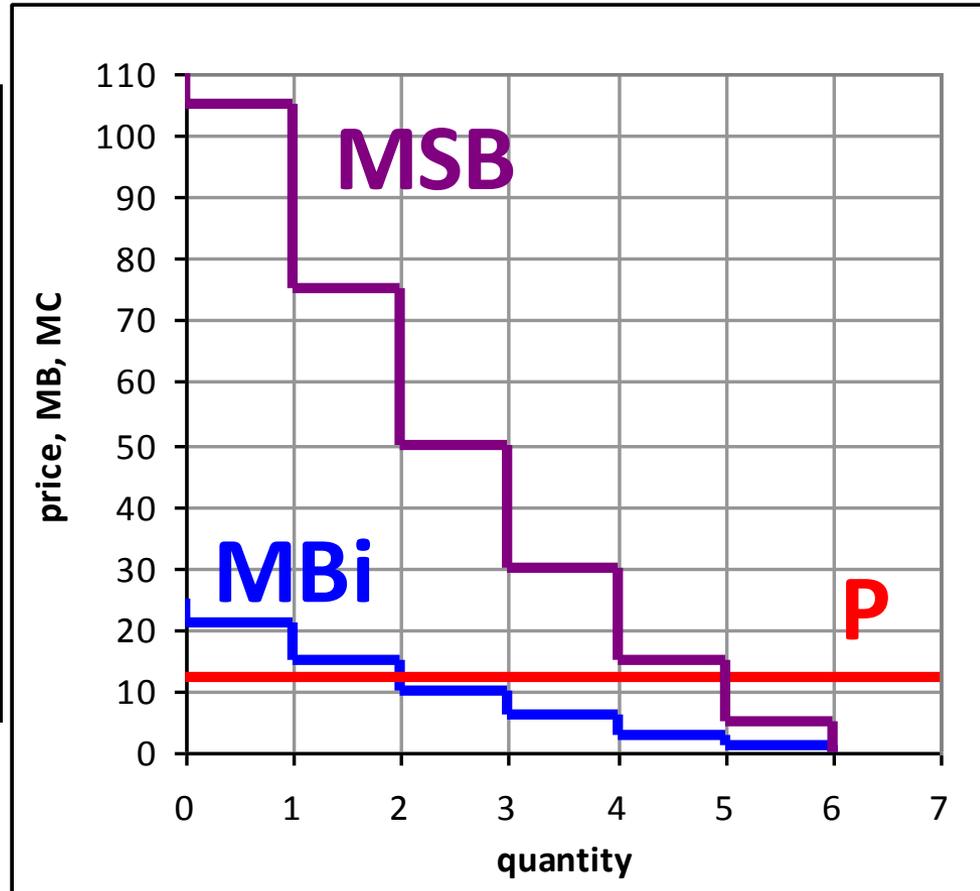
Q	TB <sub>i</sub>	MB <sub>i</sub>	TSB	MSB
1	21	21	105	105
2	36	15	180	75
3	46	10	230	50
4	52	6	260	30
5	55	3	275	15
6	56	1	280	5
7	56	0	280	0

Suppose (for simplicity) that all 5 residents of the house have exactly the same benefit-for-paintings schedule as I do. Suppose also that all paintings in the common room are non-rival and non-excludable (which makes sense).

In that case, multiplying the total benefit that each individual gets from any given number of paintings by 5 will give us a measure of the total social benefit that we get collectively. Likewise, multiplying the marginal benefit by 5 will give a measure of the marginal social benefit that each additional painting produces.

# SOCIAL BENEFIT: GRAPH

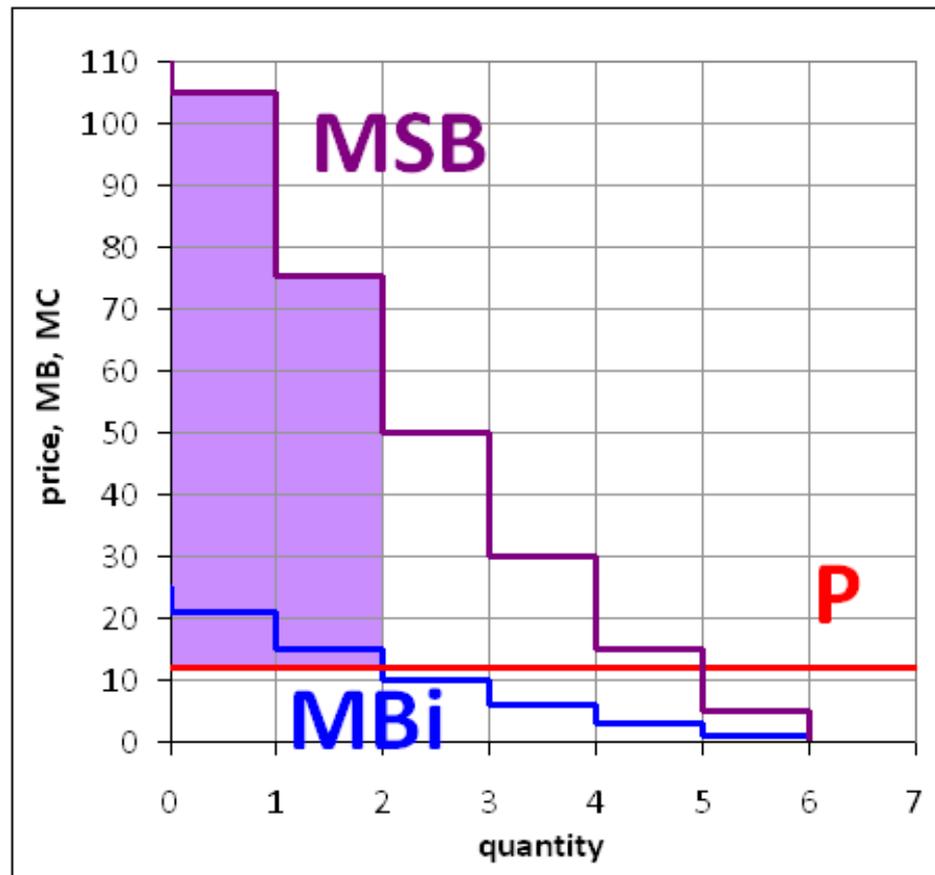
Q	TB <sub>i</sub>	MB <sub>i</sub>	TSB	MSB
1	21	21	105	105
2	36	15	180	75
3	46	10	230	50
4	52	6	260	30
5	55	3	275	15
6	56	1	280	5
7	56	0	280	0



**Non-counting question: If all of us are completely self-interested, and incapable of any kind of collective bargaining, then how many paintings will end up being bought for the house? A) 0 B) 1 C) 2 D) 3 E) 5**

# SOCIAL BENEFIT AND PRIVATE DEMAND

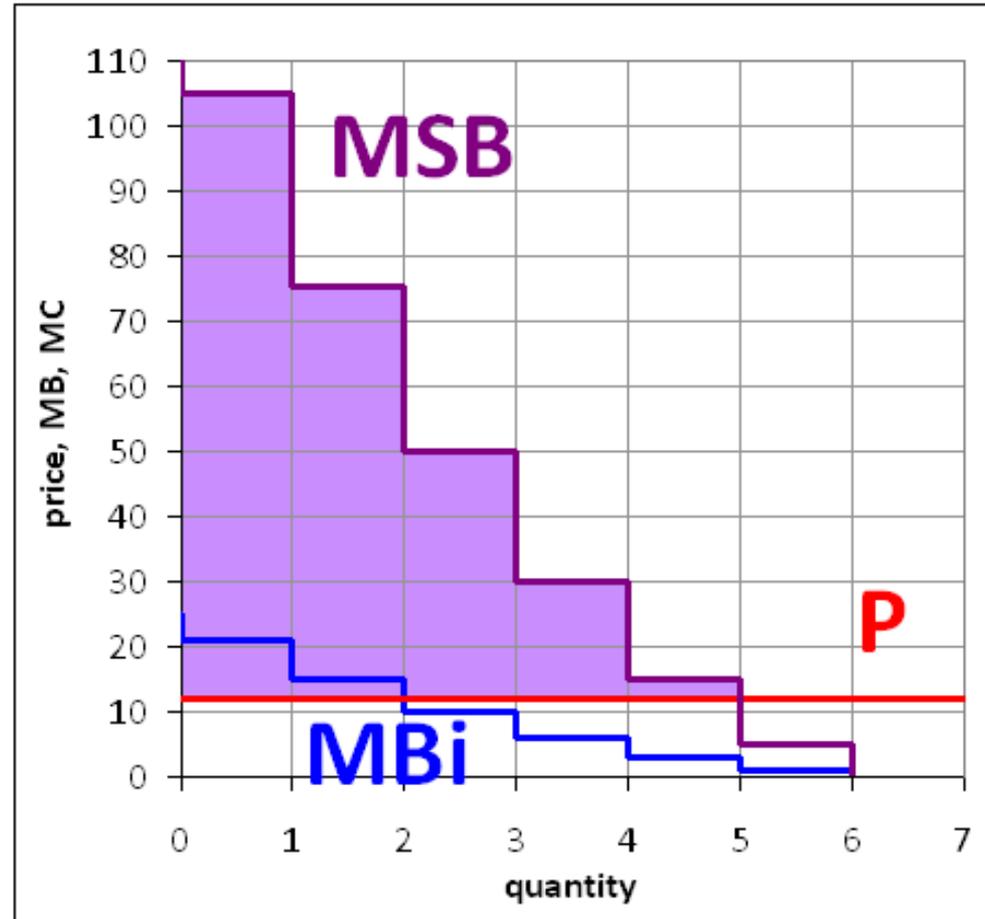
Q	TB <sub>i</sub>	MB <sub>i</sub>	TSB	MSB
1	21	21	105	105
2	36	15	180	75
3	46	10	230	50
4	52	6	260	30
5	55	3	275	15
6	56	1	280	5
7	56	0	280	0



When the number of paintings is less than 2, then it is in *someone's* private interest to buy an additional painting. However, the private marginal benefit of the 3<sup>rd</sup> painting for anyone (which is \$10) is less than the marginal cost (\$12), so only 2 will be bought. The purple area shows the total consumer surplus for everyone in the house combined, i.e. the total social benefit minus the cost of the paintings, or  $TSB(2) - 2 \times 12 = 156$ .

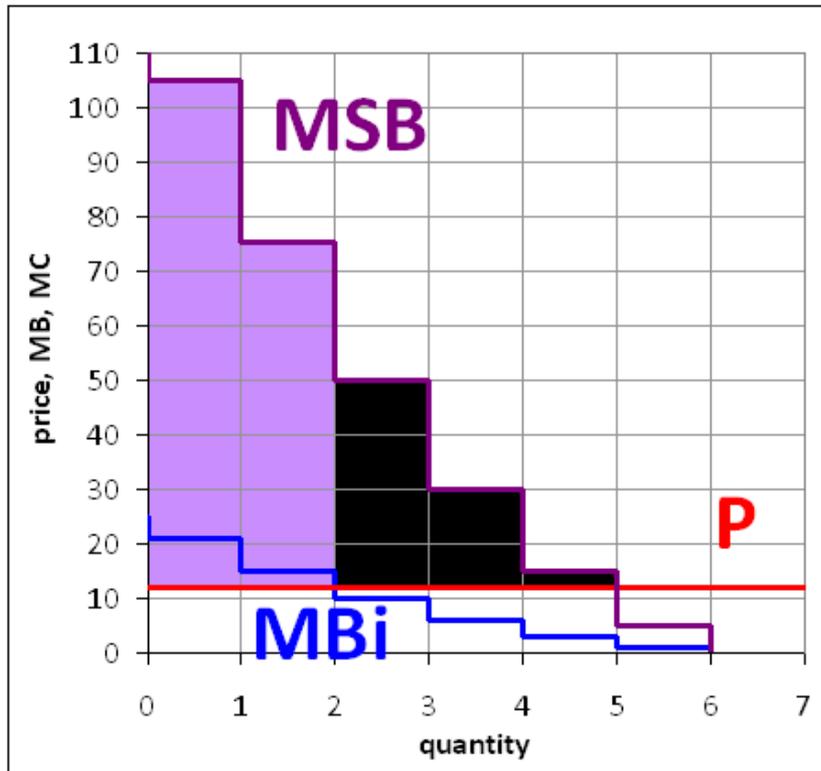
# SOCIALLY OPTIMAL PROVISION

Q	TB <sub>i</sub>	MB <sub>i</sub>	TSB	MSB
1	21	21	105	105
2	36	15	180	75
3	46	10	230	50
4	52	6	260	30
5	55	3	275	15
6	56	1	280	5
7	56	0	280	0



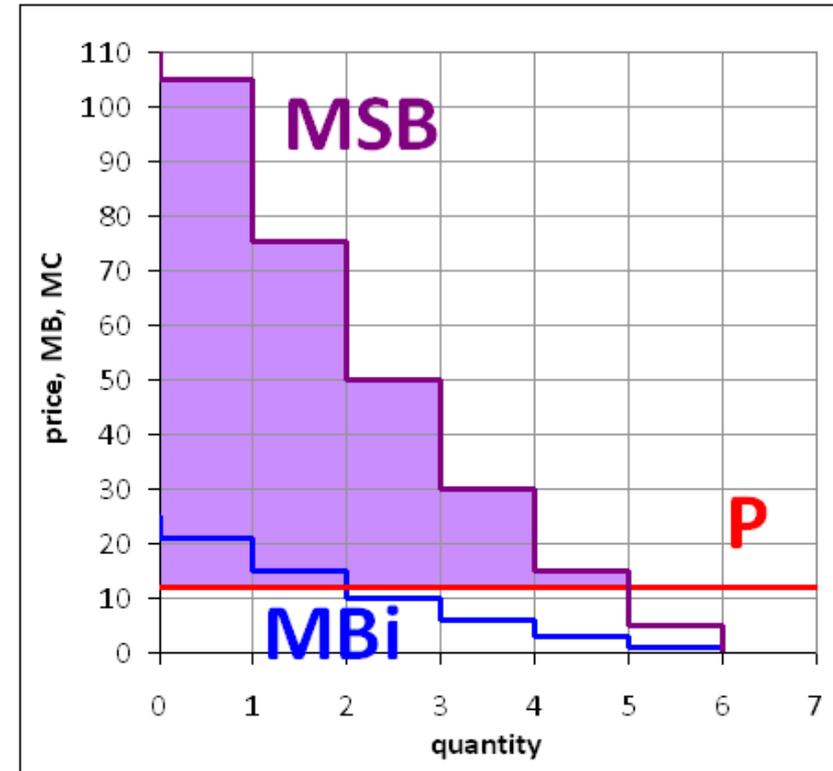
The socially optimal choice of paintings occurs where the marginal social benefit intersects the marginal cost. All paintings have a marginal cost of **12**. The 5<sup>th</sup> paintings has a marginal social benefit of **15**, and the 6<sup>th</sup> has a marginal benefit of **5**. So, only the first 5 are worth buying.

# GAIN FROM COLLECTIVE ACTION



**TSB**

105
180
230
260
275
280
280



**TSB = 180**

**TC = 2 × 12 = 24**

**TES = 180 - 24 = 156**

**TSB = 275**

**TC = 5 × 12 = 60**

**TES = 275 - 60 = 215**

**Thus, the gain from collective action (or the deadweight loss from the lack of collective action) is**  
**215 - 156 = 59.**

## QUESTION 1 (individual demand)

Q	TB <sub>i</sub>	MB <sub>i</sub>
1	13	13
2	21	8
3	26	5
4	29	3
5	31	2
6	32	1
7	32	0

Suppose that I live in a society of 10 people. My individual benefit from a public good is given in the table to the left. (TB = total benefit; MB = marginal benefit). If the price of the good is \$20, and no one else has bought or provided any of the good, how many will I choose to buy? Assume that my choice is based solely on self interest, and there is no possibility for collective agreements.

A) 0

B) 1

C) 2

D) 3

E) 4

## answer to question 1

Q	TB <sub>i</sub>	MB <sub>i</sub>
1	13	13
2	21	8
3	26	5
4	29	3
5	31	2
6	32	1
7	32	0

My individual marginal benefit is never greater than **\$20** (the price or marginal cost of the good), so it is not in my individual interest to buy any at all.

A) 0

B) 1

C) 2

D) 3

E) 4

## QUESTION 2 (marginal social benefit)

Q	TB <sub>i</sub>	MB <sub>i</sub>
1	13	13
2	21	8
3	26	5
4	29	3
5	31	2
6	32	1
7	32	0

Again there are **10** people in this society, and each has a total benefit and marginal benefit schedule for the public good as given in the table.

If 1 unit of the public good has already been provided, then what is the marginal social benefit of the second unit?

A) 0

B) 21

C) 8

D) 210

E) 80

## answer to question 2

<b>Q</b>	<b>TB<sub>i</sub></b>	<b>MB<sub>i</sub></b>	<b>TSB</b>	<b>MSB</b>
<b>1</b>	<b>13</b>	<b>13</b>	<b>130</b>	<b>130</b>
<b>2</b>	<b>21</b>	<b>8</b>	<b>210</b>	<b>80</b>
<b>3</b>	<b>26</b>	<b>5</b>	<b>260</b>	<b>50</b>
<b>4</b>	<b>29</b>	<b>3</b>	<b>290</b>	<b>30</b>
<b>5</b>	<b>31</b>	<b>2</b>	<b>310</b>	<b>20</b>
<b>6</b>	<b>32</b>	<b>1</b>	<b>320</b>	<b>10</b>
<b>7</b>	<b>32</b>	<b>0</b>	<b>320</b>	<b>0</b>

**What is the marginal social benefit of the second unit?**

**A) 0**

**B) 21**

**C) 8**

**D) 210**

**E) 80**

### QUESTION 3 (optimal quantity)

Q	TB <sub>i</sub>	MB <sub>i</sub>	TSB	MSB
1	13	13	130	130
2	21	8	210	80
3	26	5	260	50
4	29	3	290	30
5	31	2	310	20
6	32	1	320	10
7	32	0	320	0

If the marginal cost of the public good is **\$40**, then what is the socially optimal quantity of the public good?

A) 1

B) 2

C) 3

D) 4

E) 5

### answer to question 3

Q	TB <sub>i</sub>	MB <sub>i</sub>	TSB	MSB
1	13	13	130	130
2	21	8	210	80
3	26	5	260	50
4	29	3	290	30
5	31	2	310	20
6	32	1	320	10
7	32	0	320	0

If the marginal cost of the public good is **\$40**, then what is the socially optimal quantity of the public good?

A) 1

B) 2

C) 3

D) 4

E) 5

## QUESTION 4 (total surplus)

Q	TB <sub>i</sub>	MB <sub>i</sub>	TSB	MSB
1	13	13	130	130
2	21	8	210	80
3	26	5	260	50
4	29	3	290	30
5	31	2	310	20
6	32	1	320	10
7	32	0	320	0

If the marginal cost of the public good is **\$40**, and the optimal quantity of 3 is chosen, then what is the total economic surplus (total social benefit minus total cost)?

A) 140

B) 260

C) 26

D) 50

E) 40

## answer to question 4

Q	TB <sub>i</sub>	MB <sub>i</sub>	TSB	MSB
1	13	13	130	130
2	21	8	210	80
3	26	5	260	50
4	29	3	290	30
5	31	2	310	20
6	32	1	320	10
7	32	0	320	0

$$MC = 40$$

$$Q = 3$$

$$\begin{aligned} TES &= TSB - TC \\ &= 260 - 3 \times 40 \\ &= 140 \end{aligned}$$

or

$$\begin{aligned} TES &= (130 - 40) \\ &+ (80 - 40) + (50 - 40) \\ &= 90 + 40 + 10 = 140 \end{aligned}$$

**A) 140**

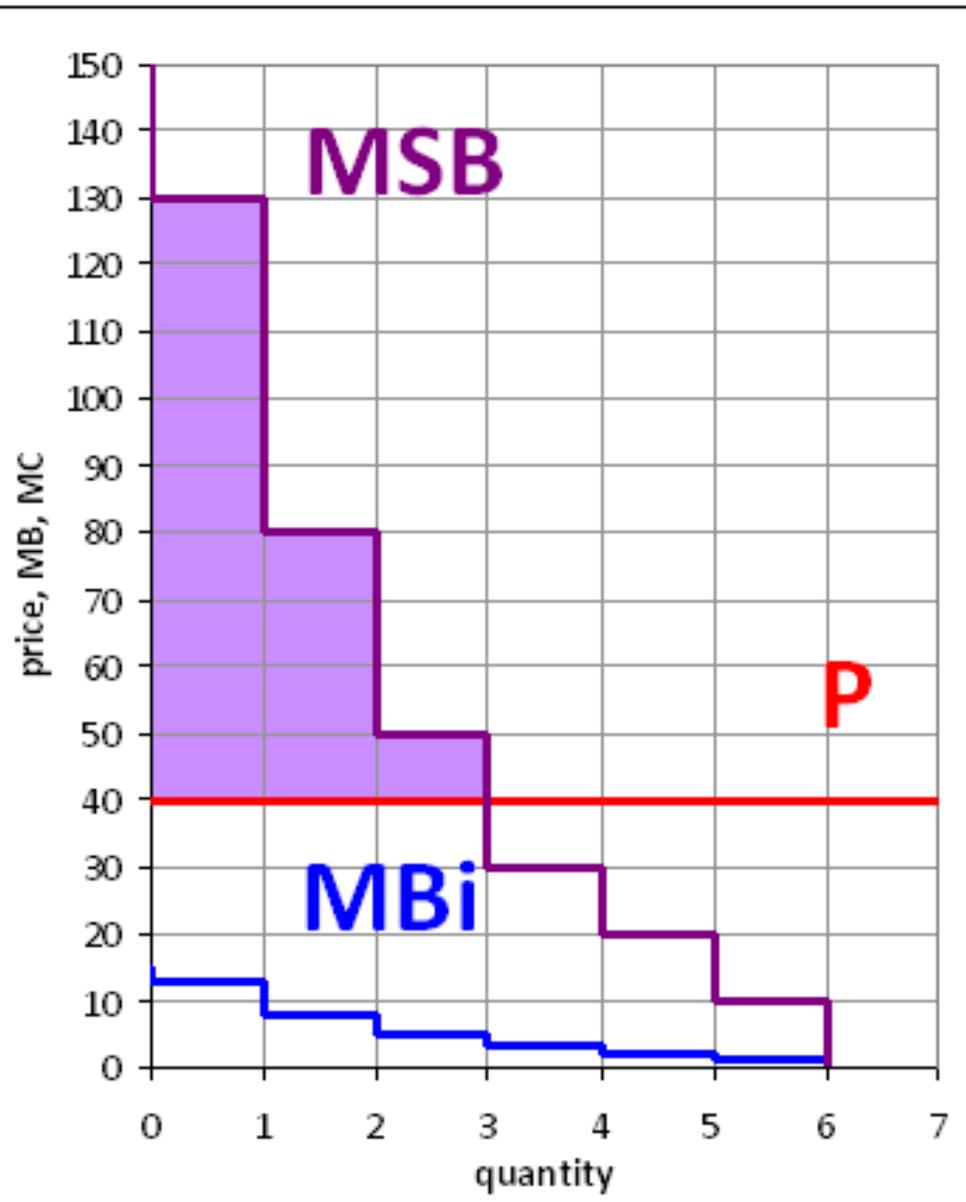
**B) 260**

**C) 26**

**D) 50**

**E) 40**

# GAIN FROM COLLECTIVE ACTION: GRAPH



**In the absence of collective action, the quantity of the public good is zero, and so the total economic surplus from the public good is also zero.**

**The optimal quantity of the public good is 3, which brings total economic surplus to 140. This is the gain from collective action, or the deadweight loss from its absence.**

## **PUBLIC GOODS: CONTINUOUS**

**Suppose that, in some society, individual total and marginal benefits from a public good are given by the functions:**

$$\mathbf{TB_i = 100Q - Q^2}$$

$$\mathbf{MB_i = 100 - 2Q}$$

**If there are 5 people in the society, then the social total and marginal benefit functions are:**

$$\mathbf{TSB = 500Q - 5Q^2}$$

$$\mathbf{MSB = 500 - 10Q}$$

# PRIVATE DEMAND

$$TB_i = 100Q - Q^2$$

$$MB_i = 100 - 2Q$$

$$TSB = 500Q - 5Q^2$$

$$MSB = 500 - 10Q$$

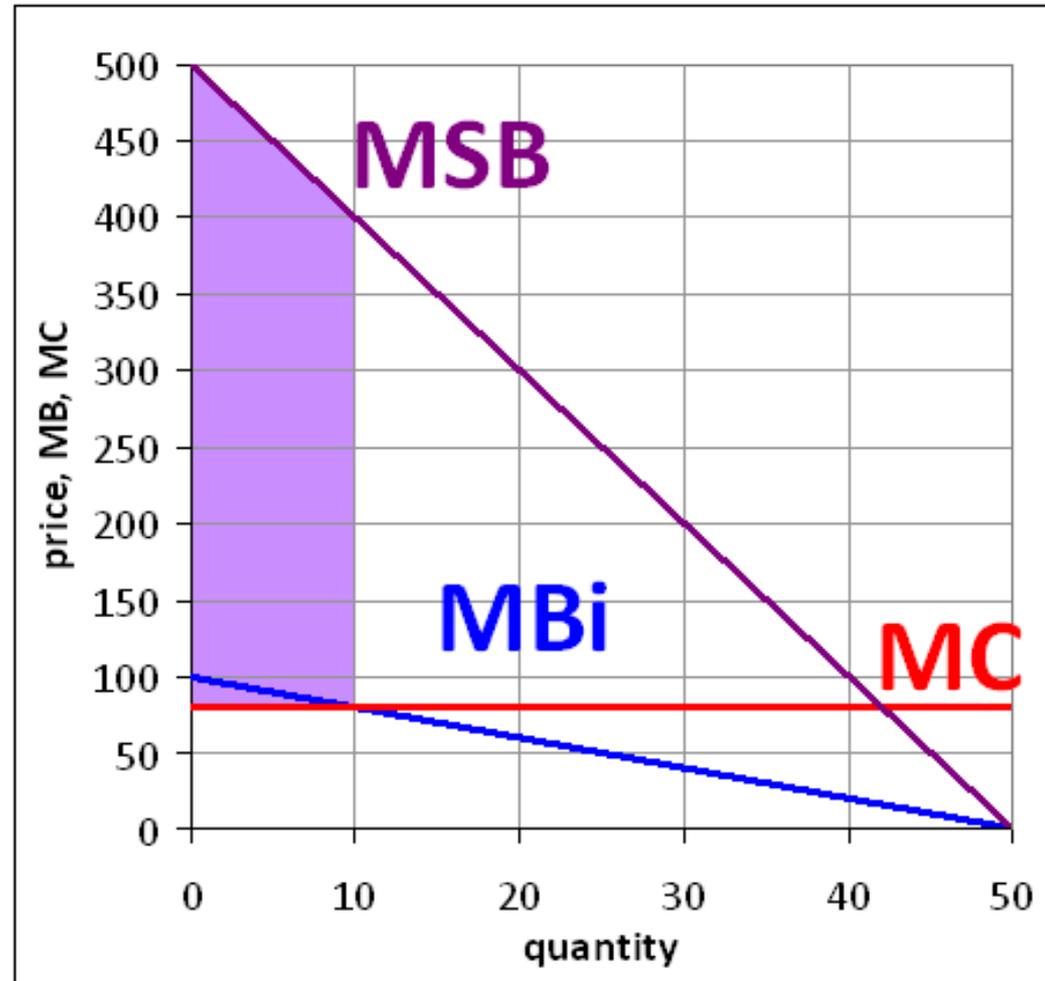
$$MC = 80$$

If there is no collective action, then people will only buy the public good up to the point where  $MB_i = MC$

$$100 - 2Q = 80$$

$$2Q = 20$$

$Q^* = 10$  is the equilibrium quantity.



# SOCIALLY OPTIMAL PROVISION

$$TB_i = 100Q - Q^2$$

$$MB_i = 100 - 2Q$$

$$TSB = 500Q - 5Q^2$$

$$MSB = 500 - 10Q$$

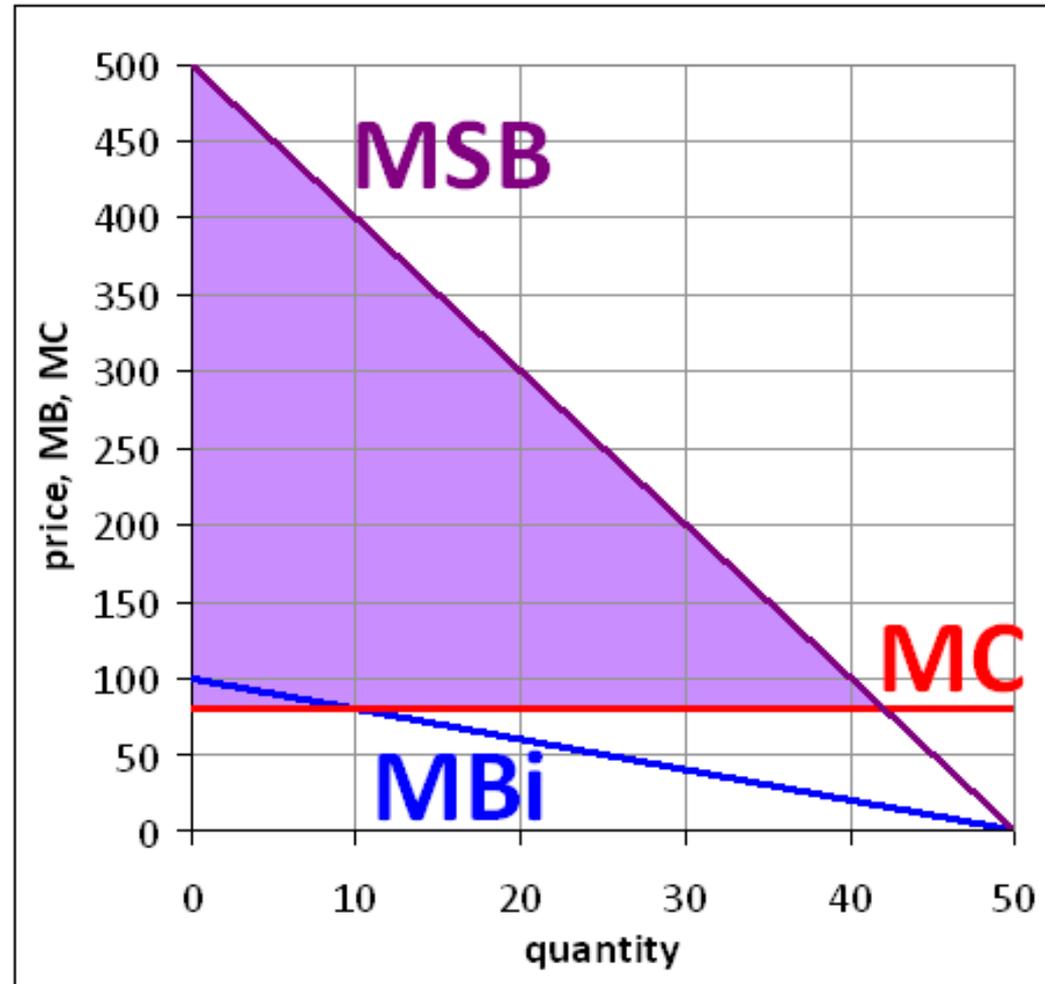
$$MC = 80$$

**Total economic surplus from the public good will be maximized at the point where  $MSB = MC$**

$$500 - 10Q = 80$$

$$10Q = 420$$

**$Q^o = 42$  is the socially optimal quantity.**



# GAIN FROM COLLECTIVE ACTION (GEOMETRIC)

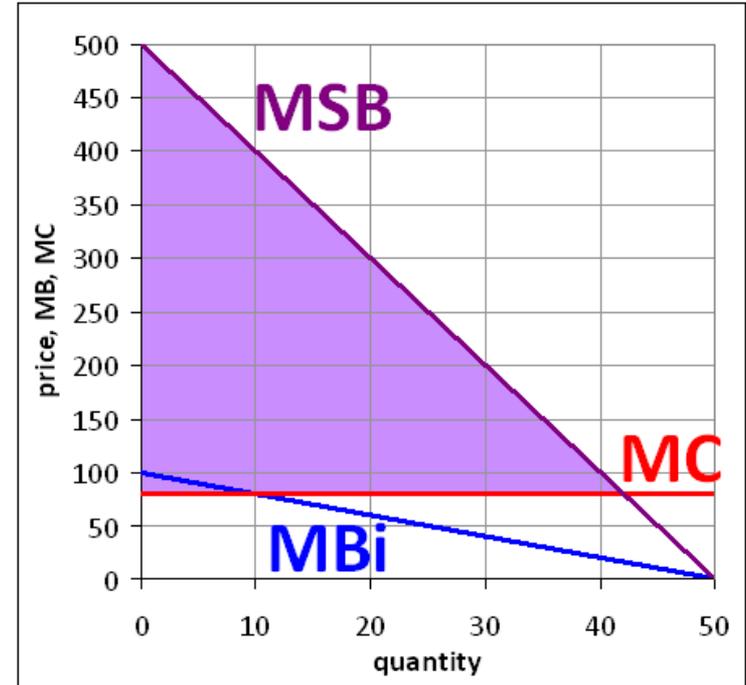
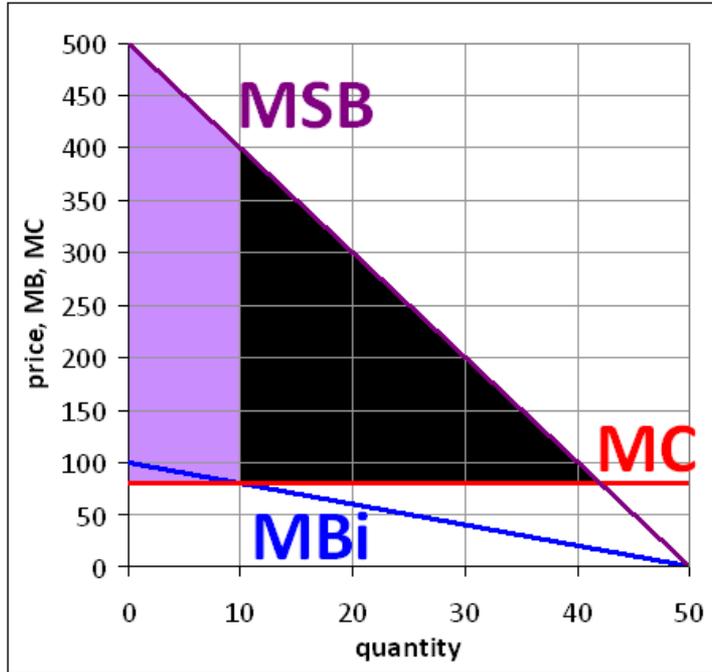
$$TB_i = 100Q - Q^2$$

$$MB_i = 100 - 2Q$$

$$MC = 80$$

$$TSB = 500Q - 5Q^2$$

$$MSB = 500 - 10Q$$



$$TES = (10)(420+320)/2$$

$$= 3700$$

$$TES = (.5)(42)(420)$$

$$= 8820$$

$$DWL = (.5)(32)(320) = 5120$$

# GAIN FROM COLLECTIVE ACTION (USING TSB)

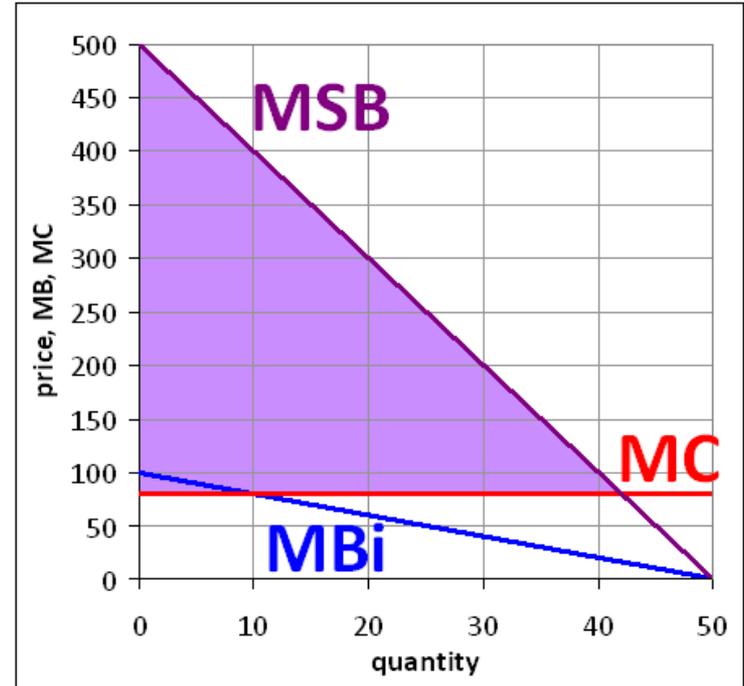
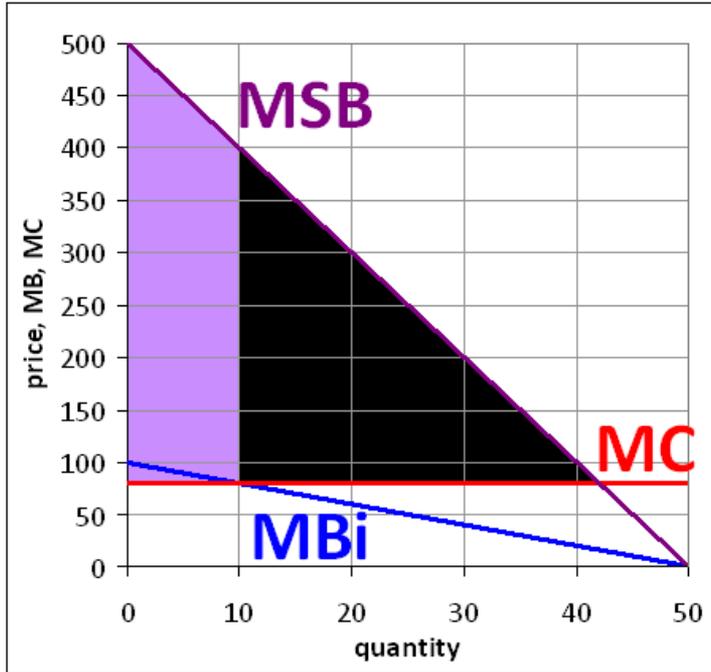
$$TB_i = 100Q - Q^2$$

$$MB_i = 100 - 2Q$$

$$MC = 80$$

$$TSB = 500Q - 5Q^2$$

$$MSB = 500 - 10Q$$



$$\begin{aligned} TSB &= 500(10) - 5(10)^2 \\ &= 5000 - 500 = 4500 \end{aligned}$$

$$TC = 10 \times 80 = 800$$

$$TES = 4500 - 800 = 3700$$

$$\begin{aligned} TSB &= 500(42) - 5(42)^2 \\ &= 21000 - 8820 = 12180 \end{aligned}$$

$$TC = 42 \times 80 = 3360$$

$$TES = 12180 - 3360 = 8820$$

$$DWL = 8820 - 3700 = 5120$$

## QUESTION 5

$$TB_i = 15Q - Q^2/20$$

$$MB_i = 15 - Q/10$$

$$TSB = 150Q - Q^2/2$$

$$MSB = 150 - Q \quad (10 \text{ people})$$

$$MC = 10$$

**Total and marginal benefit functions for a public good for a society of 10 identical people are given above, along with the price marginal cost of the public good (10).**

**If there is no possibility for collective action, how much of the public good will be bought?**

**A) 0**

**B) 20**

**C) 40**

**D) 50**

**E) 100**

## answer to question 5

$$TB_i = 15Q - Q^2/20$$

$$MB_i = 15 - Q/10$$

$$TSB = 150Q - Q^2/2$$

$$MSB = 150 - Q \quad (10 \text{ people})$$

$$MC = 10$$

$$MB_i = MC \rightarrow 15 - Q/10 = 10 \rightarrow Q/10 = 5$$

$$\rightarrow Q = 50$$

A) 0

B) 20

C) 40

D) 50

E) 100

## QUESTION 6

$$TB_i = 15Q - Q^2/20$$

$$MB_i = 15 - Q/10$$

$$TSB = 150Q - Q^2/2$$

$$MSB = 150 - Q \quad (10 \text{ people})$$

$$MC = 20$$

Same question, but the marginal cost is now **20**.

If there is no possibility for collective action, how much of the public good will be bought?

A) 0

B) 20

C) 40

D) 50

E) 100

## answer to question 6

$$TB_i = 15Q - Q^2/20$$

$$MB_i = 15 - Q/10$$

$$TSB = 150Q - Q^2/2$$

$$MSB = 150 - Q \quad (10 \text{ people})$$

$$MC = 20$$

Same question, but the marginal cost is now **20**.

If there is no possibility for collective action, how much of the public good will be bought?

**A) 0**

**B) 20**

**C) 40**

**D) 50**

**E) 100**

## QUESTION 7

$$TB_i = 15Q - Q^2/20$$

$$MB_i = 15 - Q/10$$

$$TSB = 150Q - Q^2/2$$

$$MSB = 150 - Q \quad (10 \text{ people})$$

$$MC = 20$$

The marginal cost is still **20**.

What is the quantity of the public good that maximizes total surplus?

A) 0

B) 50

C) 100

D) 120

E) 130

## answer to question 7

$$TB_i = 15Q - Q^2/20$$

$$MB_i = 15 - Q/10$$

$$TSB = 150Q - Q^2/2$$

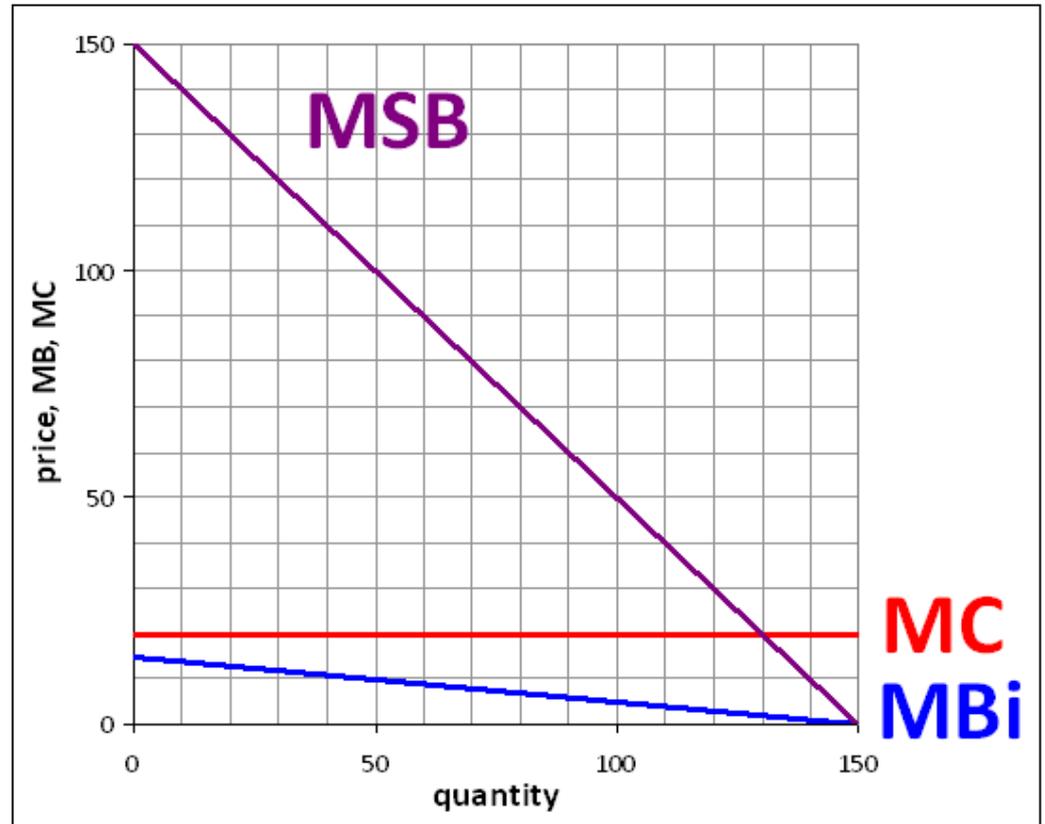
$$MSB = 150 - Q \quad (10 \text{ people})$$

$$MC = 20$$

$$MSB = MC$$

$$\rightarrow 150 - Q = 20$$

$$\rightarrow Q = 130$$



A) 0

B) 50

C) 100

D) 120

E) 130

## QUESTION 8

$$TB_i = 15Q - Q^2/20$$

$$MB_i = 15 - Q/10$$

$$TSB = 150Q - Q^2/2$$

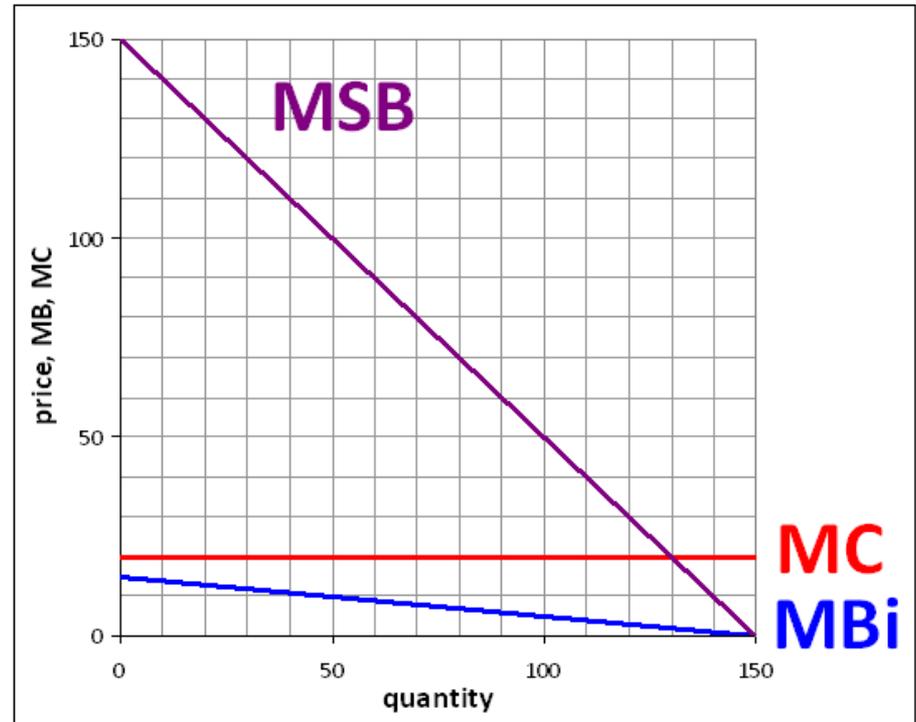
$$MSB = 150 - Q \quad (10 \text{ people})$$

$$MC = 20$$

$$Q^* = 0$$

$$Q^\circ = 130$$

If this society chooses the optimal quantity of 130, then how much economic surplus have they gained from collective action?



A) 130

B) 11250

C) 16900

D) 8450

E) 22500

## answer to question 8

$$TB_i = 15Q - Q^2/20$$

$$MB_i = 15 - Q/10$$

$$TSB = 150Q - Q^2/2$$

$$MSB = 150 - Q \quad (10 \text{ people})$$

$$MC = 20$$

$$Q^* = 0$$

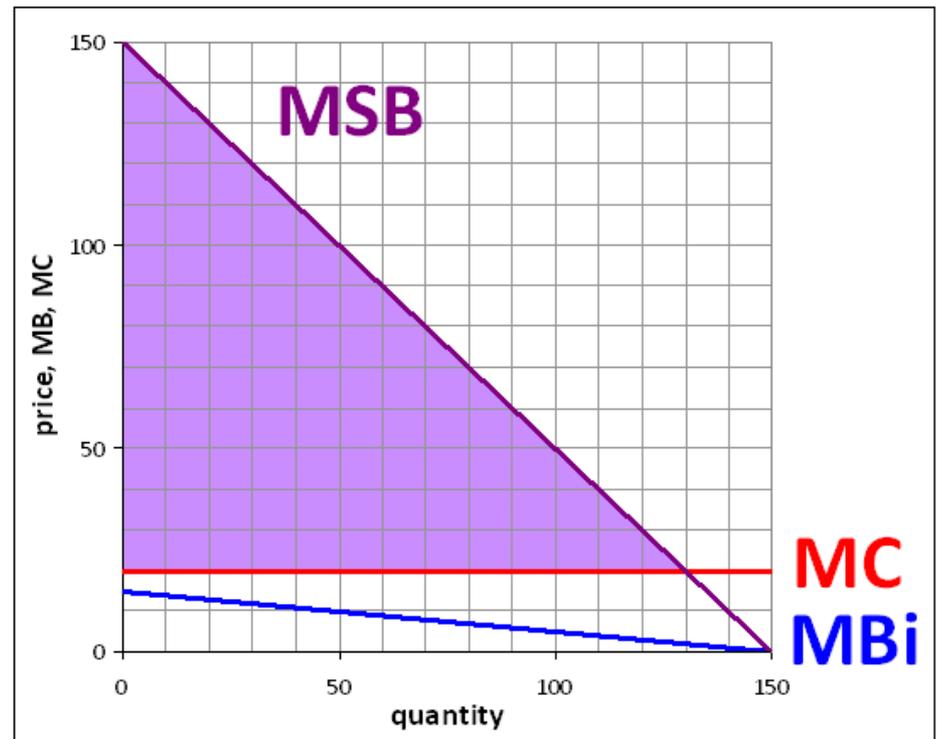
$$Q^o = 130$$

$$TES = (.5)(130)(130)$$

$$= (.5)(16900)$$

$$= 8000 + 450$$

$$TES = 8450$$



A) 130

B) 11250

C) 16900

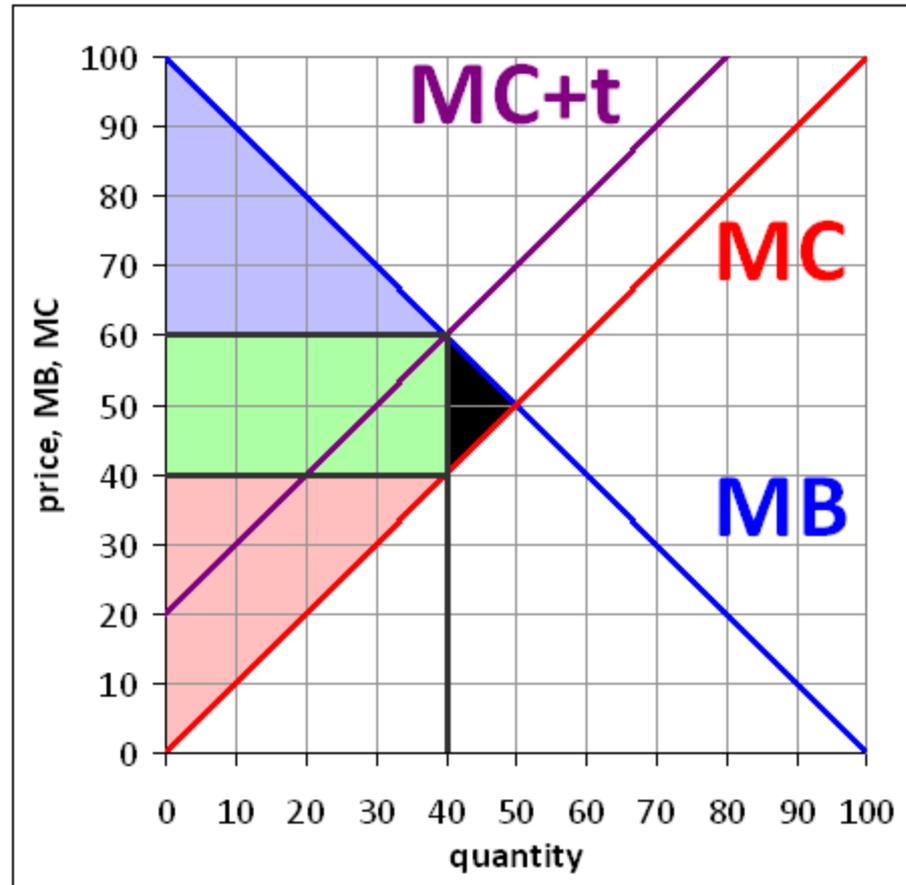
D) 8450

E) 22500

# TAXES ON EFFICIENT MARKETS

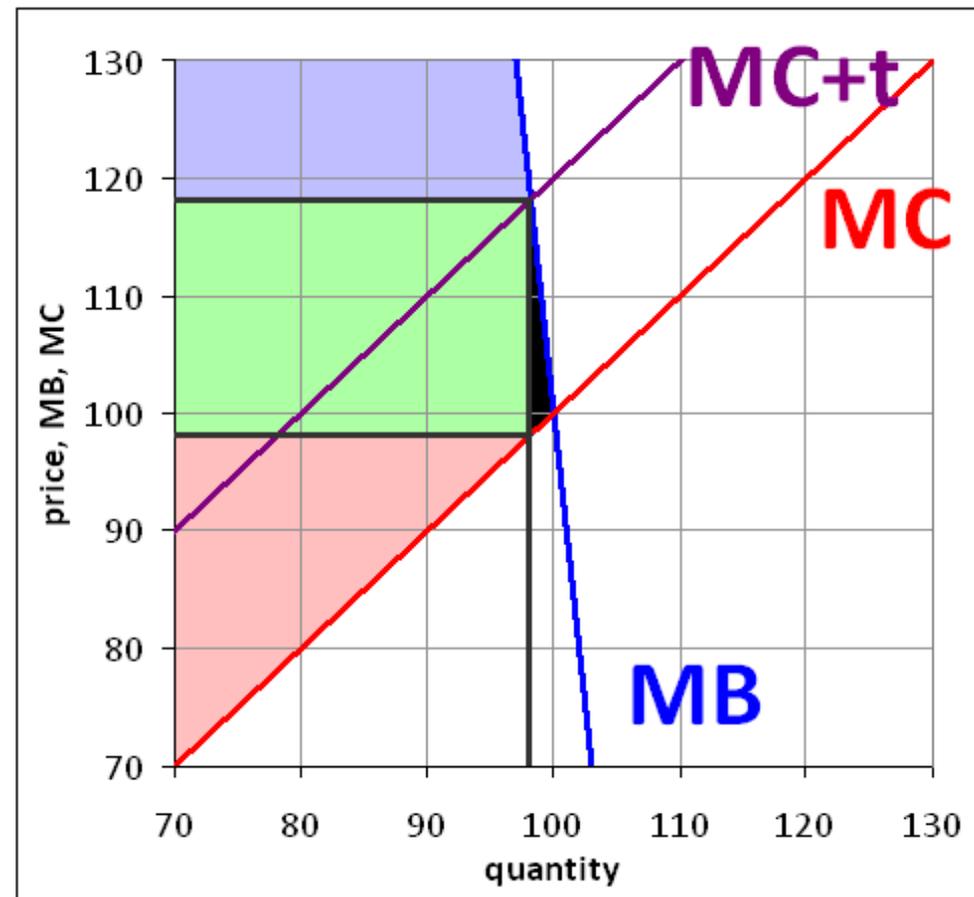
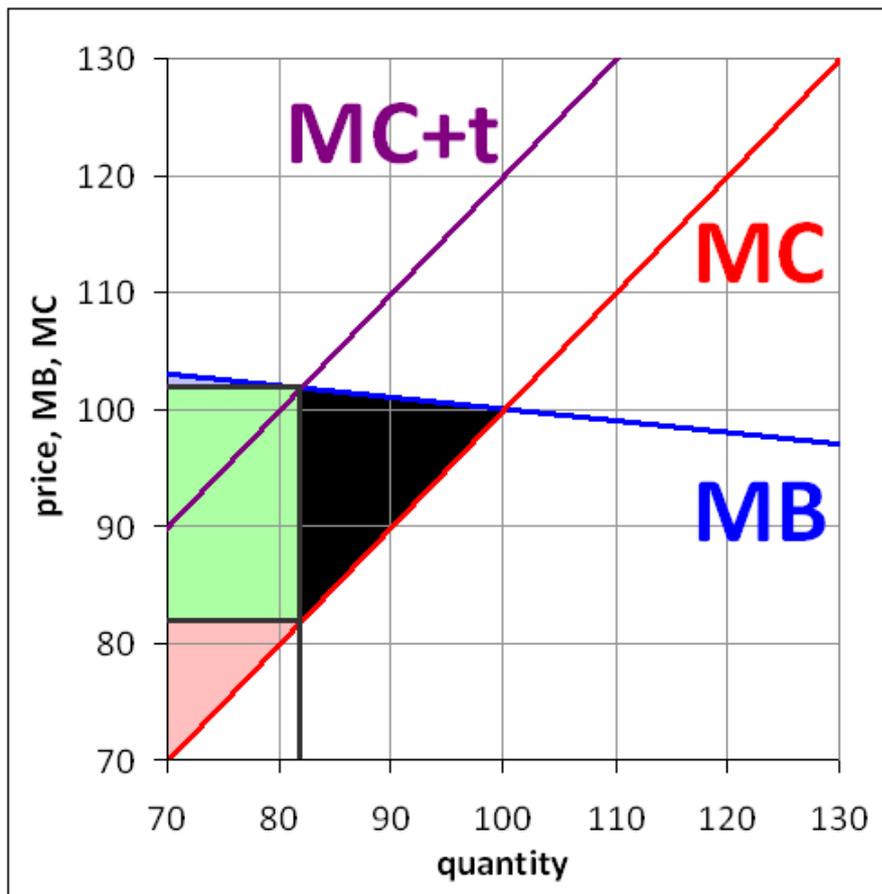
Suppose that we have an initially efficient market (perfectly competitive, with no externalities), and we apply an excise (per unit) tax.

The **blue** area shows **consumer surplus**, the **red** area shows **producer surplus**, and the **green area** shows **government revenue, G**.



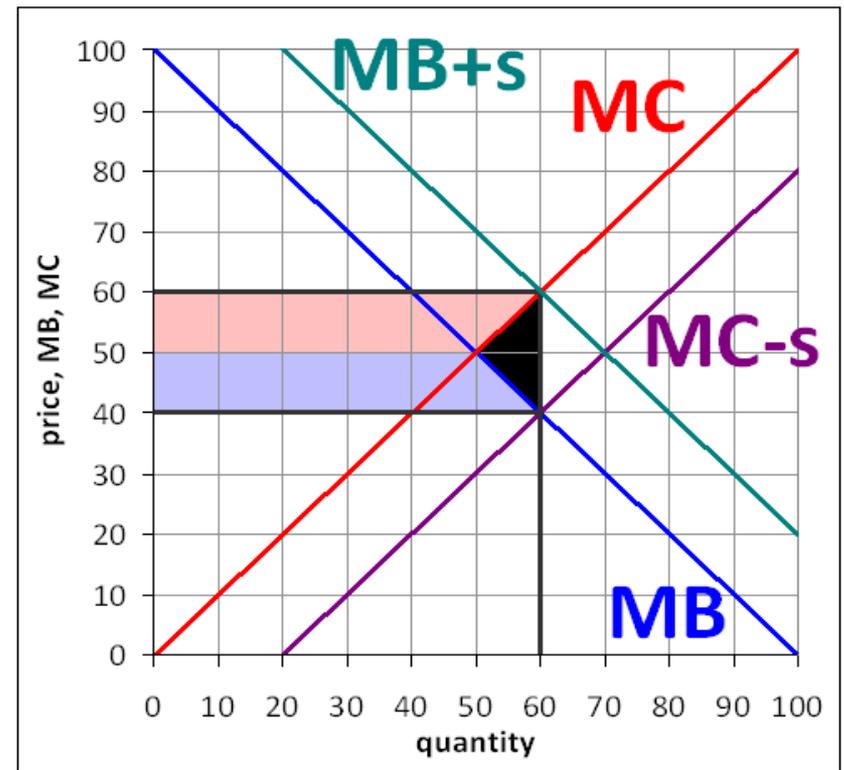
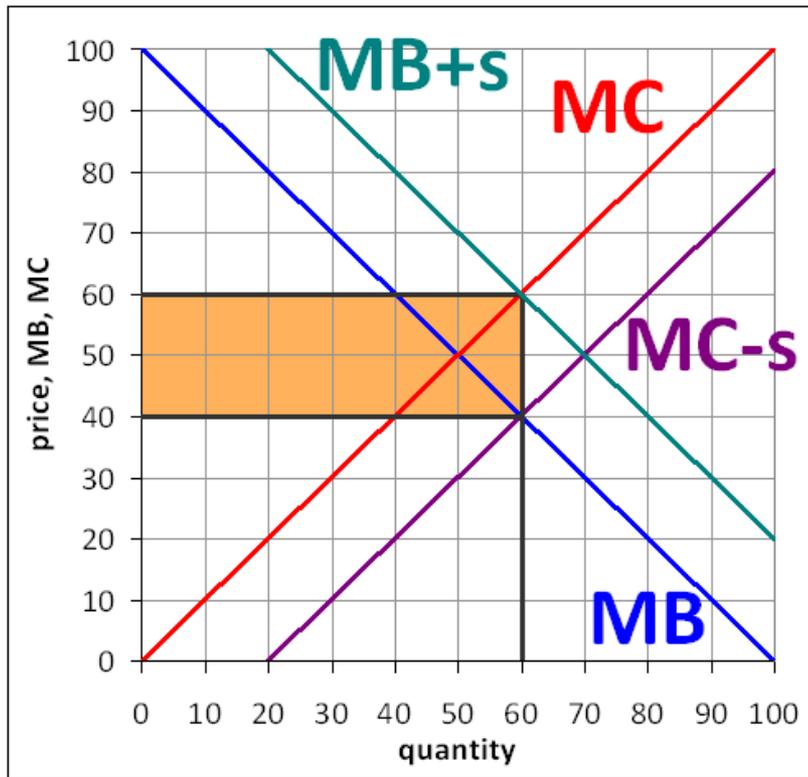
If  $t$  is the tax per unit, and  $Q$  is the quantity of the good sold, then  $G = tQ$ .

# ELASTICITY AND DEADWEIGHT LOSS



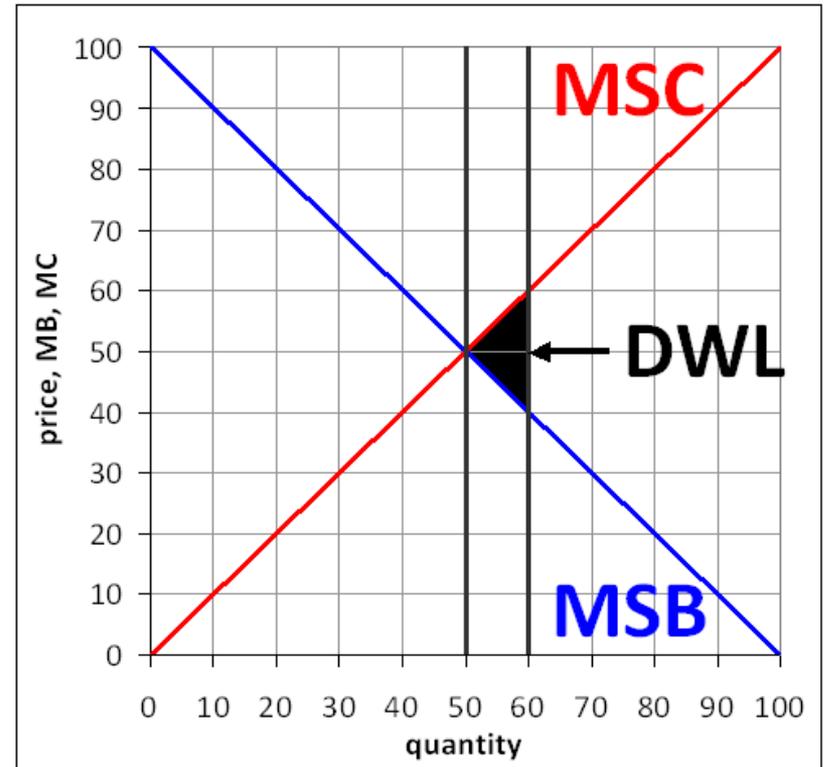
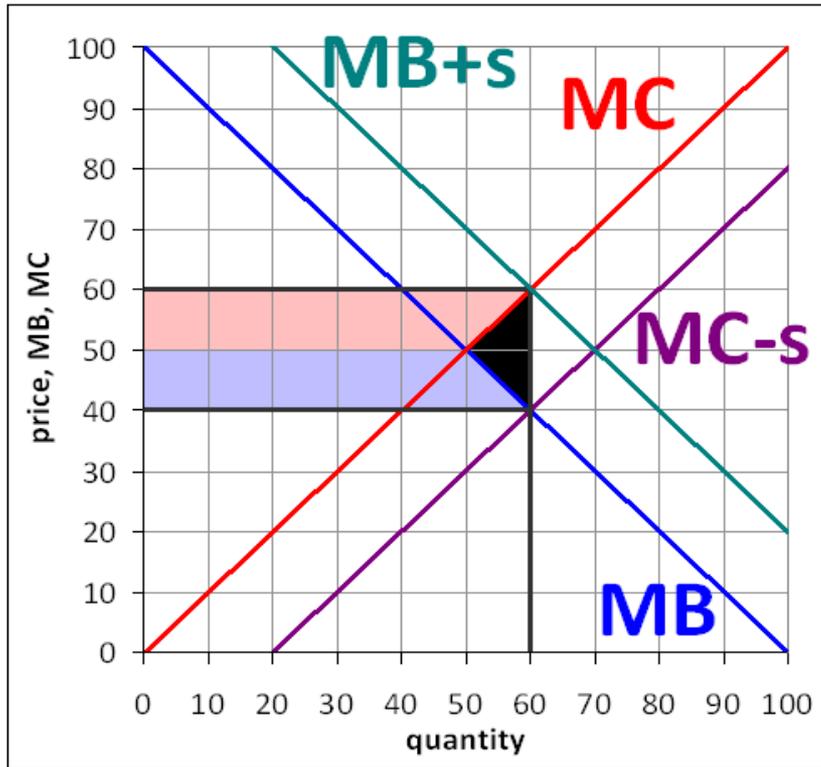
**When either the supply or demand is highly elastic (sensitive to price changes), then the deadweight loss of taxation tends to be higher, as shown on the left above. If either is perfectly inelastic, then taxation has no deadweight loss.**

# ADDING SUBSIDIES TO EFFICIENT MARKETS



**Adding a subsidy to an already-efficient market can also cause a loss in total economic surplus. Here, the orange area represents the money that the government must pay to support the subsidy, the blue area represents the gain in consumer surplus, the red area represents the gain in producer surplus, and the black area is a deadweight loss, i.e. lost government revenue that doesn't become either consumer or producer surplus.**

# ADDING SUBSIDIES TO EFFICIENT MARKETS



**Adding a subsidy to an already-efficient market decreases surplus because you are causing the market to produce when the marginal social cost is greater than the marginal social benefit. The distance between these two defines the deadweight loss.**

# TAXES: ALGEBRA

First, with no tax...

$$\text{MB} = 120 - 2Q$$

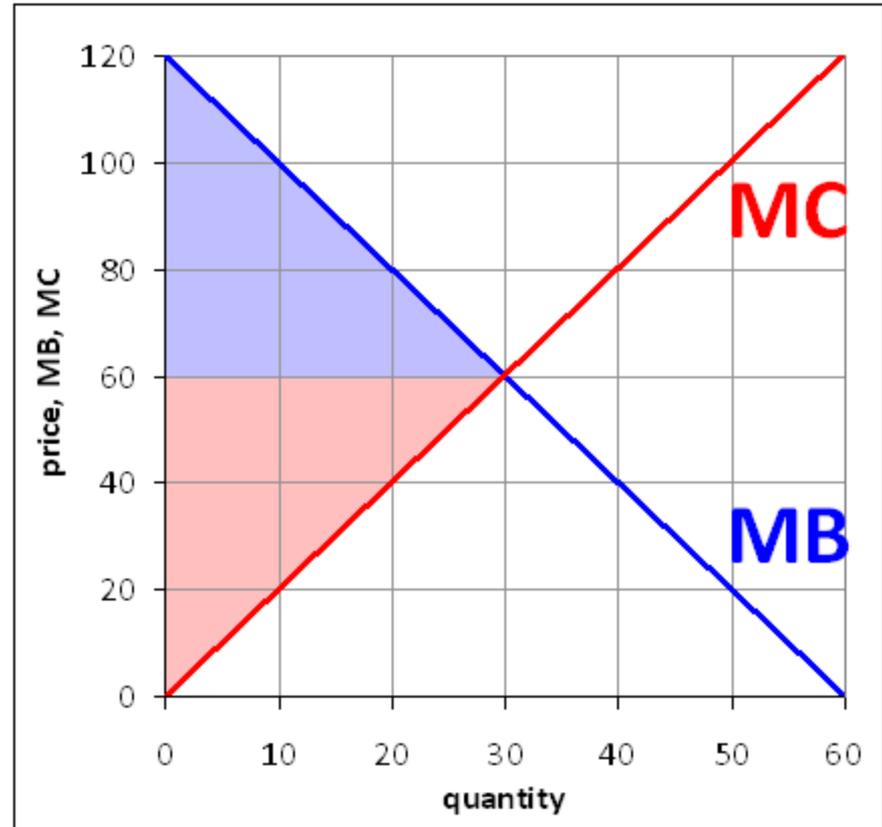
$$\text{MC} = 2Q$$

$$\begin{aligned}\text{MB} = \text{MC} &\rightarrow 120 - 2Q = 2Q \\ \rightarrow 4Q = 120 &\rightarrow Q = 30\end{aligned}$$

$$\text{CS} = (.5)(30)(60) = 900$$

$$\text{PS} = (.5)(30)(60) = 900$$

$$\text{TES} = \text{CS} + \text{PS} = 1800$$



# TAXES: ALGEBRA

$$\text{MB} = 120 - 2Q$$

$$\text{MC} = 2Q$$

$$t = 40$$

$$\text{MB} = \text{MC} + t$$

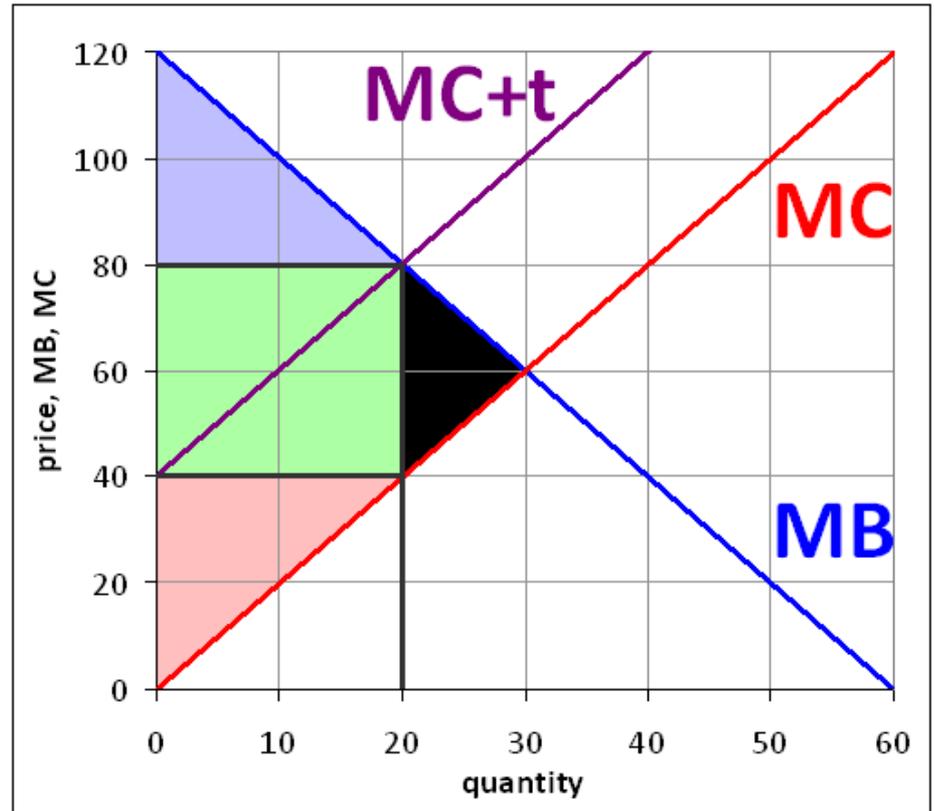
$$\rightarrow 120 - 2Q = 2Q + 40$$

$$\rightarrow 4Q = 80 \rightarrow Q = 20$$

$$\text{CS} = (.5)(20)(40) = 400$$

$$\text{PS} = (.5)(20)(40) = 400$$

$$\text{G} = tQ = (40)(20) = 800$$

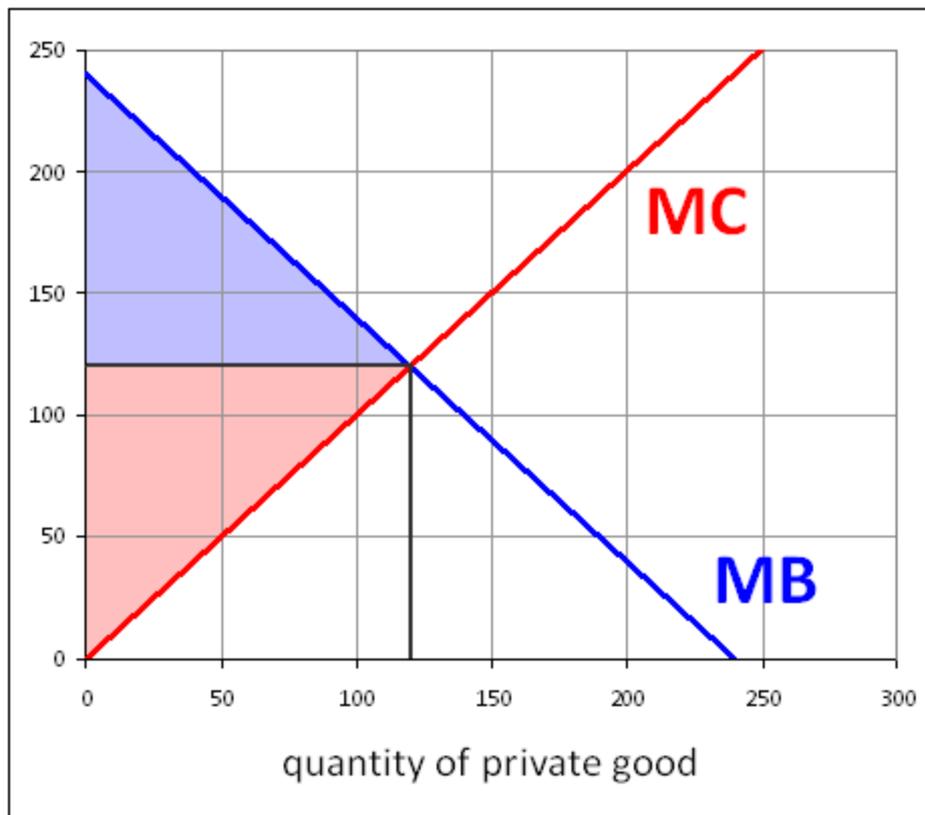


With the tax of 40,  $\text{TES} = \text{CS} + \text{PS} + \text{G} = 1600$ .

*Without the tax, TES was 1800, so  $\text{DWL} = 200$ .*

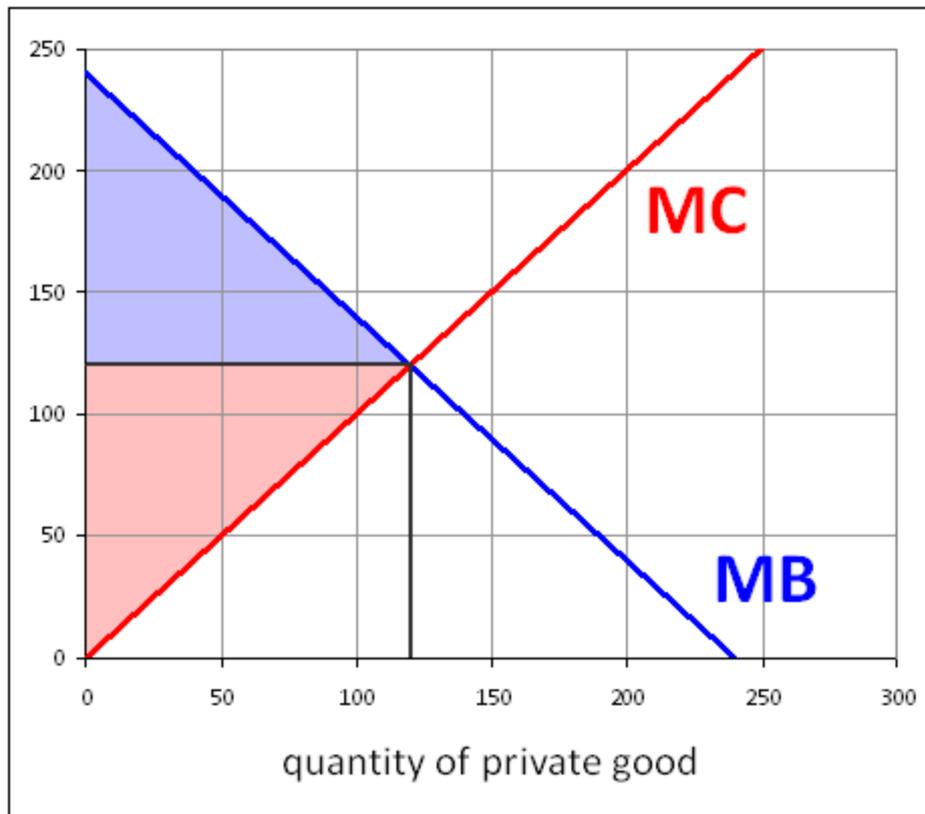
You can also find that using  $\text{DWL} = (.5)(10)(40)$ , calculating the area on the graph above.

# TWO-MARKET MODEL OF GOVERNMENT



**Suppose that there are two goods: a private good, and a public good. The market for the private good happens to be efficient (it is perfectly competitive and has no externalities), but in the absence of collective action, the equilibrium quantity of the public good is zero.**

# TWO-MARKET MODEL OF GOVERNMENT



$$\mathbf{MB = 240 - x}$$

$$\mathbf{MC = x}$$

$$\mathbf{x^* = 120}$$

$$\mathbf{CS = (.5)(120)(120) = 7200}$$

$$\mathbf{PS = (.5)(120)(120) = 7200}$$

$$\mathbf{TES = 7200 + 7200 = 14400}$$



$$\mathbf{MSB = 150 - .5y}$$

$$\mathbf{MC = 100}$$

$$\mathbf{y^* = 0}$$

$$\mathbf{CS = 0}$$

$$\mathbf{PS = 0}$$

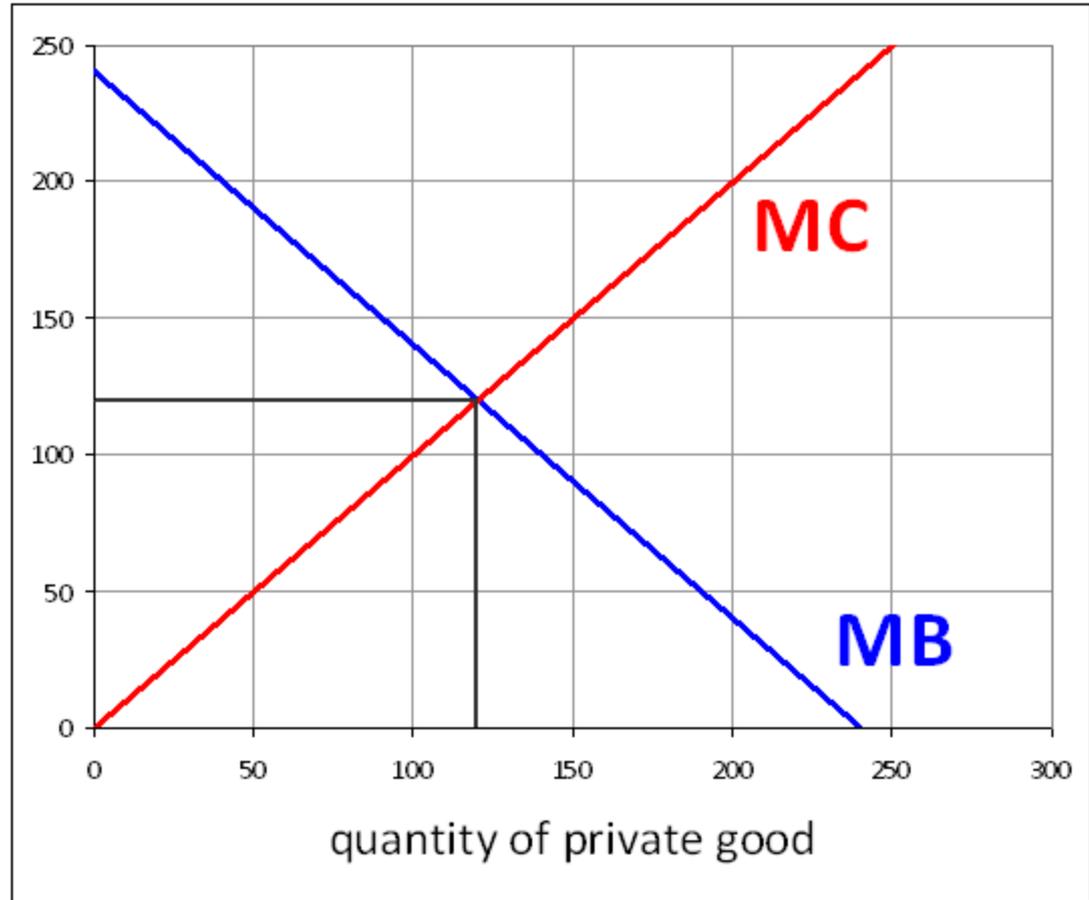
$$\mathbf{TES = 0}$$

## QUESTION 9 (tax, $\Delta Q$ )

$$MB = 240 - x$$

$$MC = x$$

If a tax of \$40 per unit is imposed, then how much of the good will be bought and sold in equilibrium?



A) 0

B) 80

C) 100

D) 120

E) 140

## answer to question 9

$$\text{MB} = 240 - x$$

$$\text{MC} = x$$

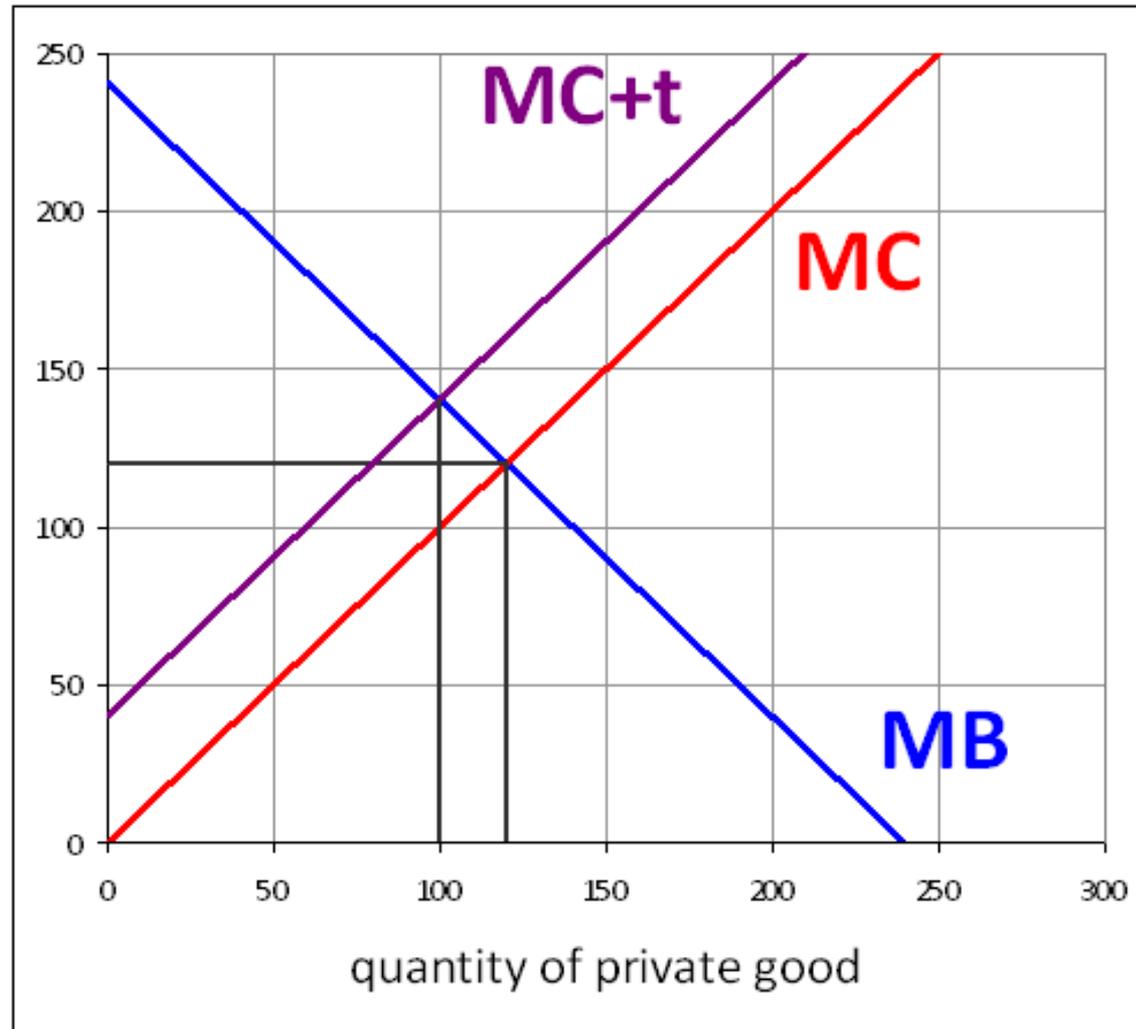
If a tax of \$40 per unit is imposed, then how much of the good will be bought and sold in equilibrium?

$$\text{MB} = \text{MC} + t$$

$$\rightarrow 240 - x = x + 40$$

$$\rightarrow 2x = 200$$

$$\rightarrow x^* = 100$$



A) 0

B) 80

C) 100

D) 120

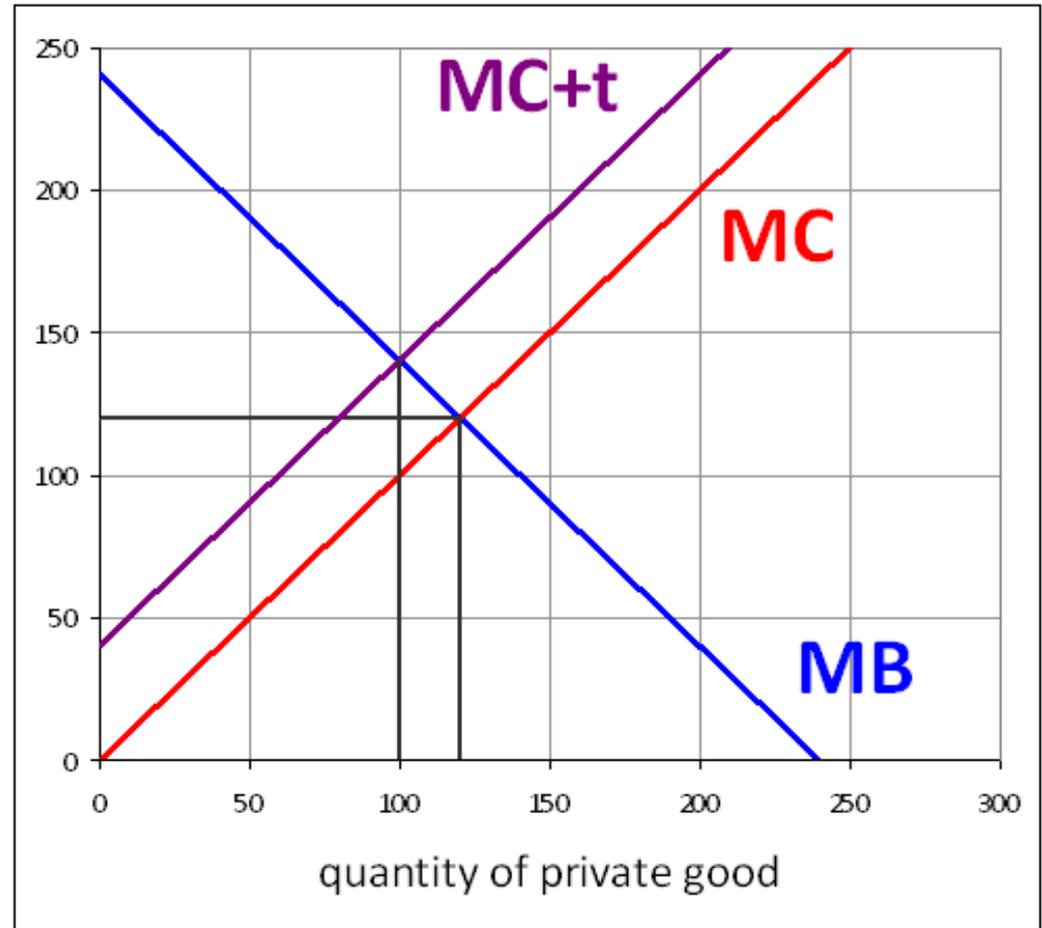
E) 140

## QUESTION 10 (tax, G)

$$MB = 240 - x$$

$$MC = x$$

If a tax of \$40 per unit is imposed, then the equilibrium quantity will change from 120 to 100. How much revenue will the government get from the tax?



A) 4000

B) 4800

C) 1000

D) 2400

E) 1400

# answer to question 10

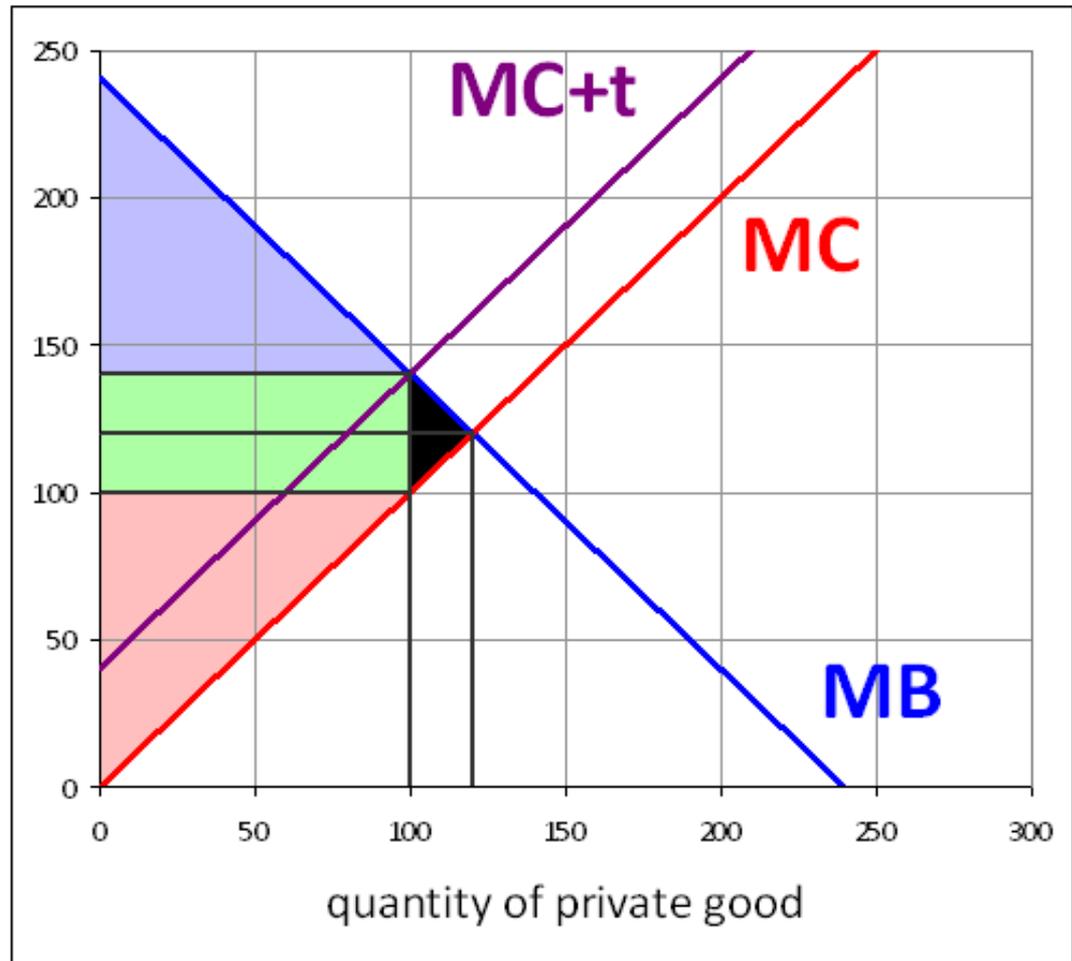
$$MB = 240 - x$$

$$MC = x$$

$$t = 40$$

$$Q = 100$$

$$G = tQ = 40 \times 100 \\ = 4000$$



**A) 4000**

**B) 4800**

**C) 1000**

**D) 2400**

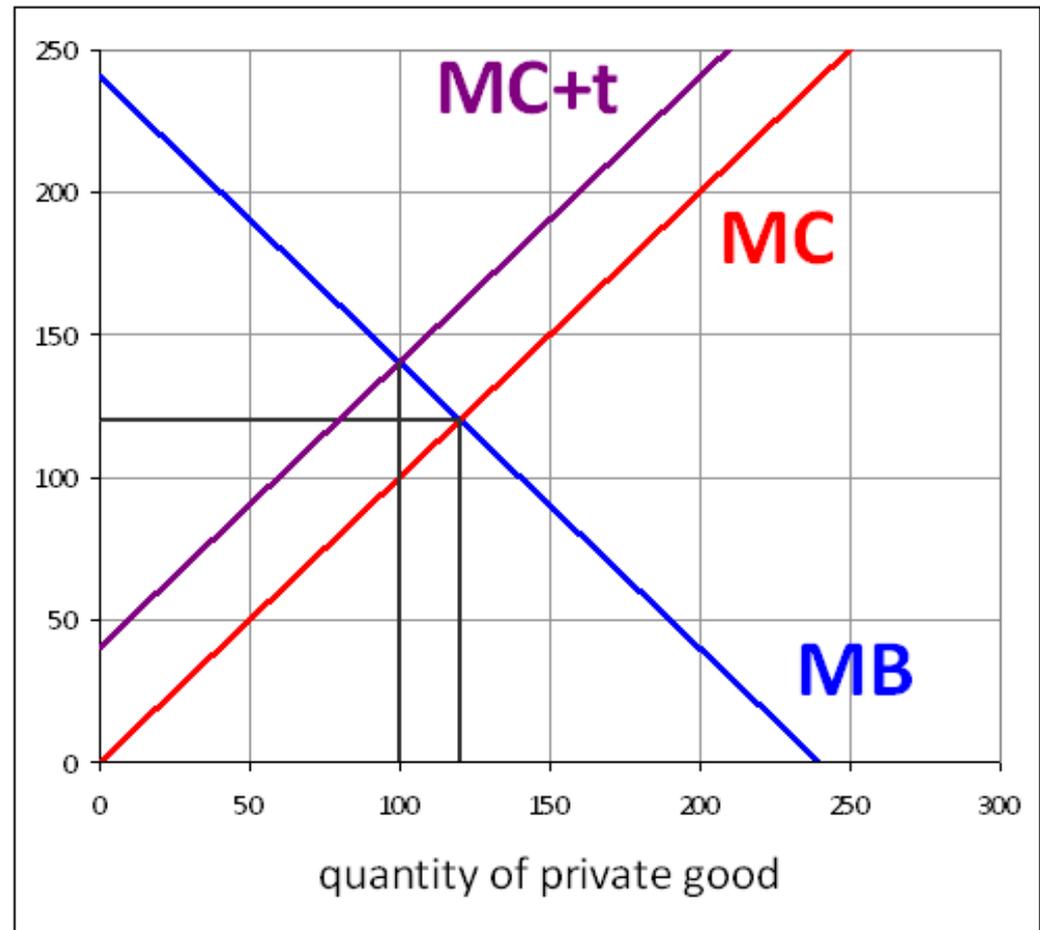
**E) 1400**

## QUESTION 11 (tax, DWL)

$$MB = 240 - x$$

$$MC = x$$

If a tax of \$40 per unit is imposed, then the equilibrium quantity will change from 120 to 100. How much **deadweight loss** will result from the tax? (That is, by how much will TES decrease in the private goods market?)



A) 400

B) 480

C) 100

D) 240

E) 140

# answer to question 11

$$MB = 240 - x$$

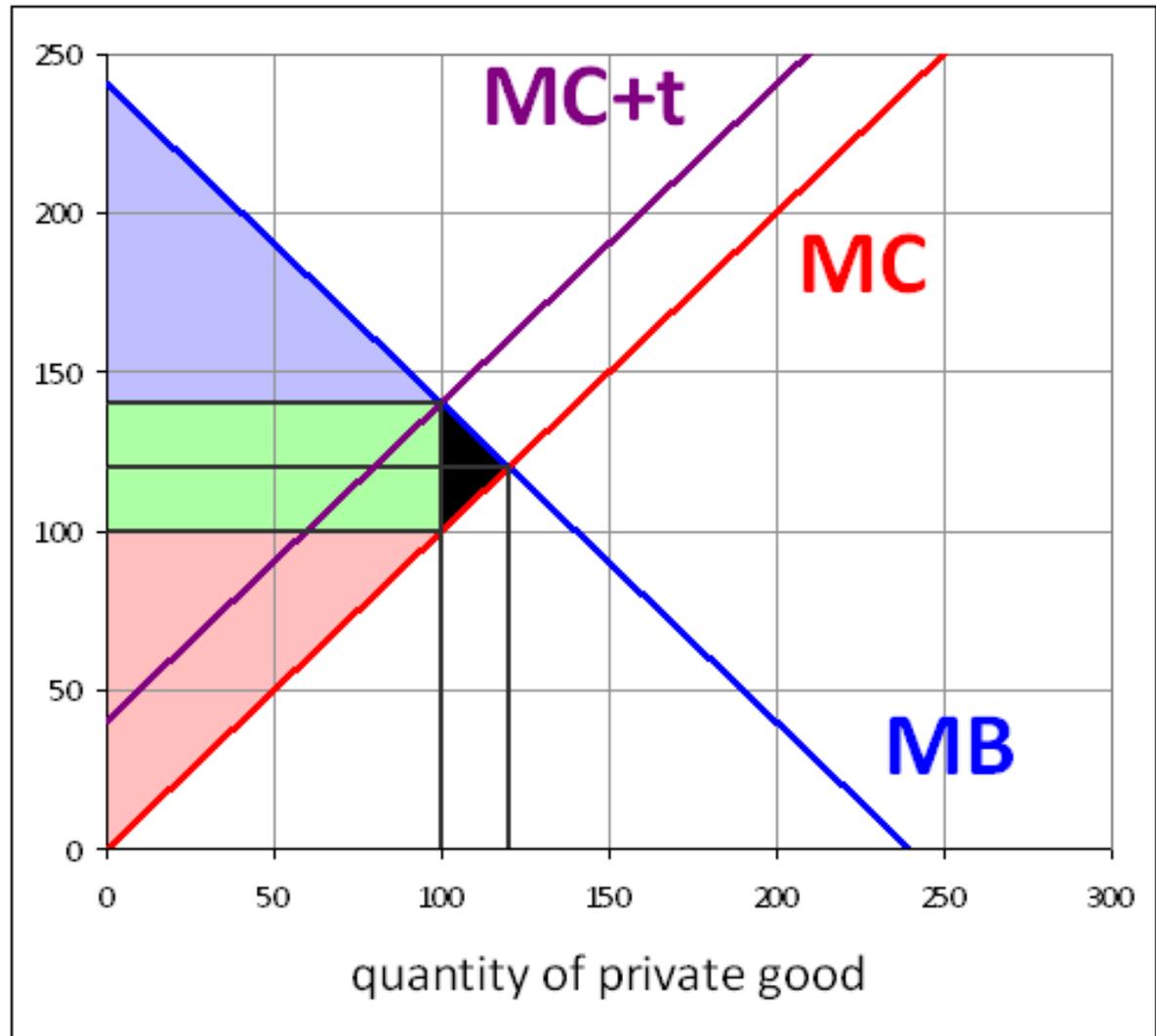
$$MC = x$$

$$t = 40$$

$\Delta Q = 20$  (changed  
from 120 to 100)

$$DWL = (.5)(20)(40)$$

$$DWL = 400$$



**A) 400**

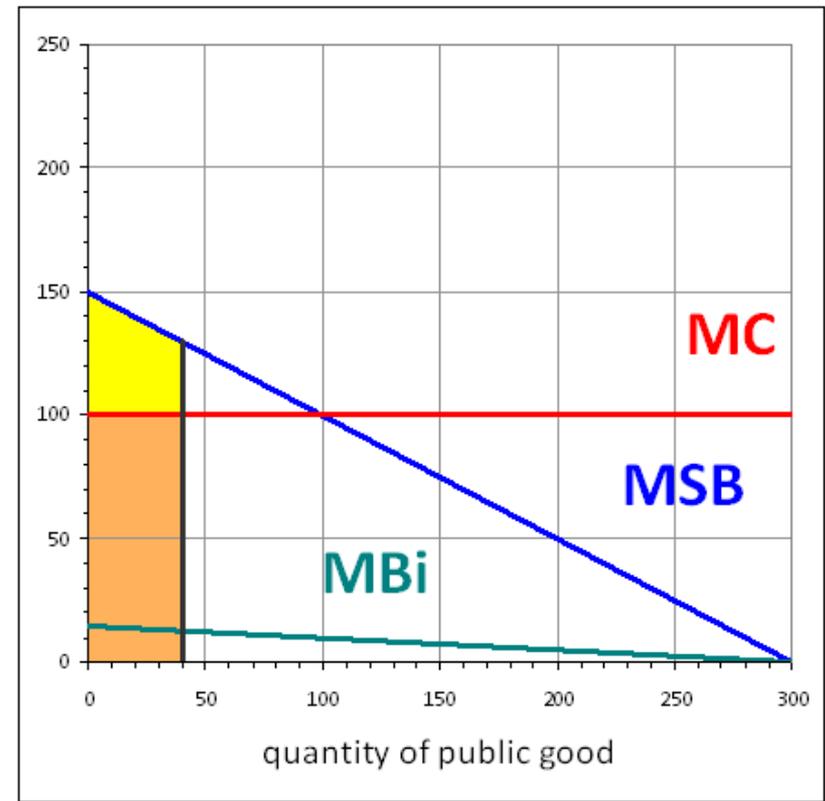
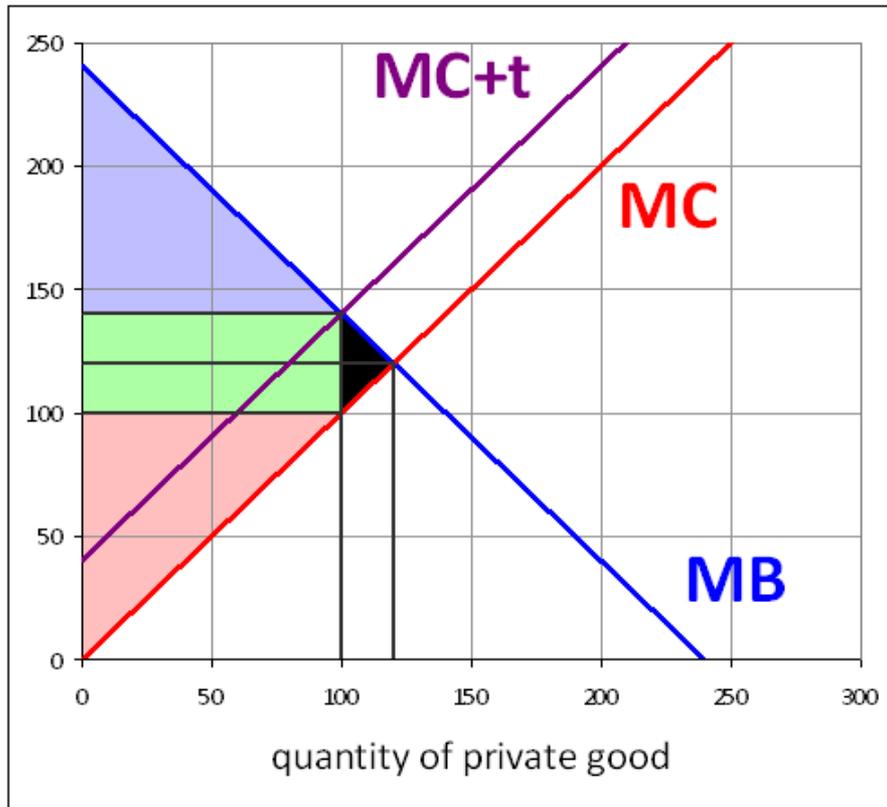
**B) 480**

**C) 100**

**D) 240**

**E) 140**

# TWO-MARKET MODEL OF GOVERNMENT



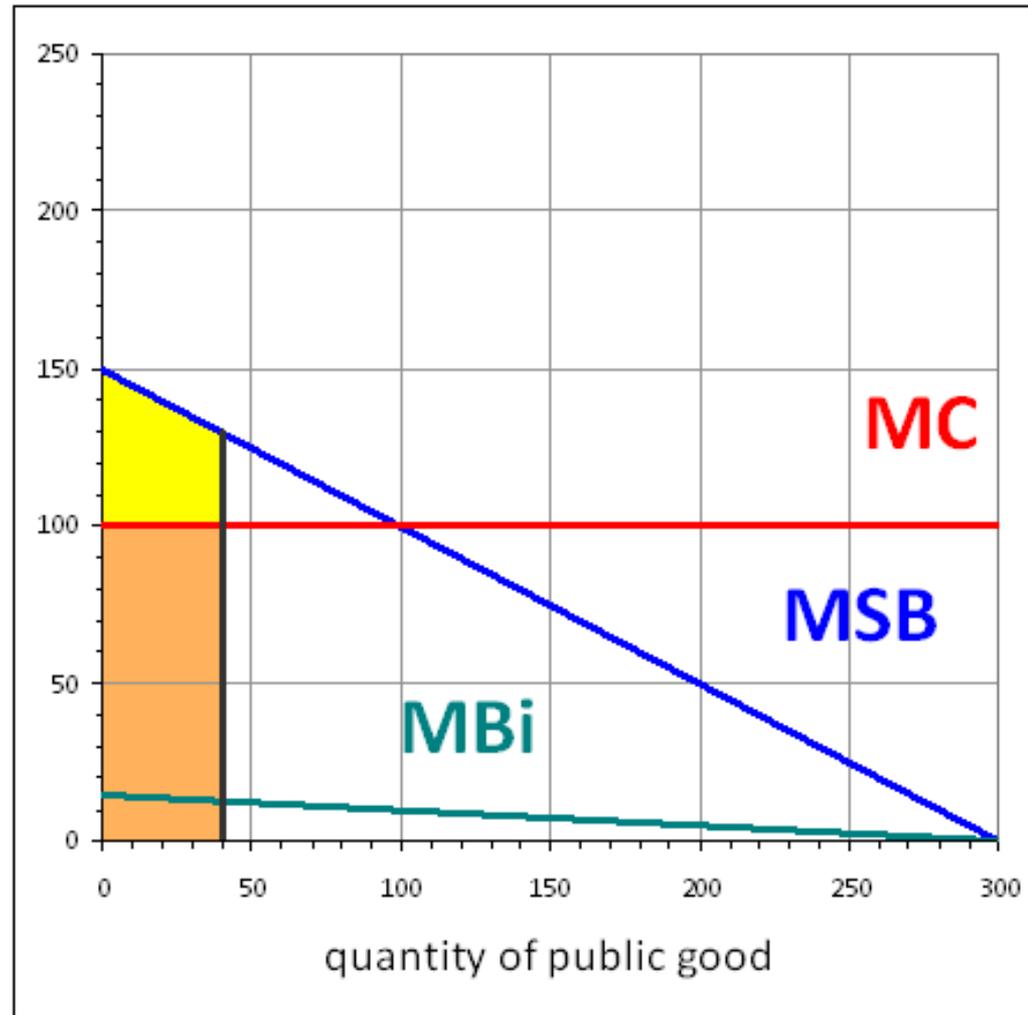
$$\begin{aligned}
 \text{MB} &= 240 - x & \text{MC} &= x \\
 t &= 40 & G &= 4000 \\
 \text{DWL} &= 400 \\
 \text{CS} &= 5000 & \text{PS} &= 5000 \\
 \text{TES} &= 4000 + 5000 + 5000 \\
 &= 14000 \text{ (down from 14400)}
 \end{aligned}$$

Suppose that the **\$4000** raised from taxes in the private goods market is spent in the public goods market...

# TWO-MARKET MODEL OF GOVERNMENT

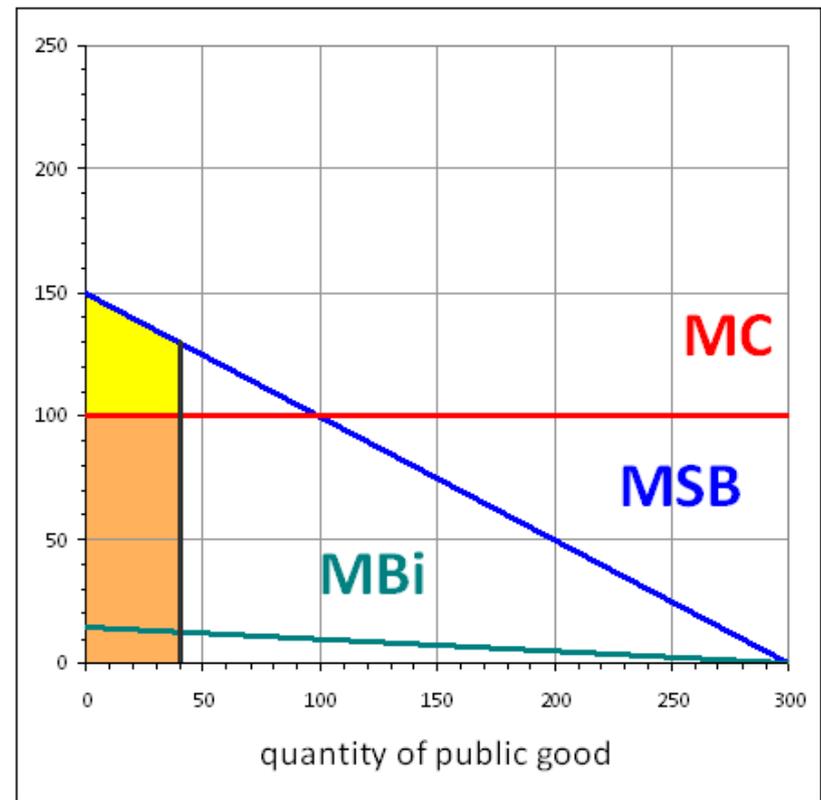
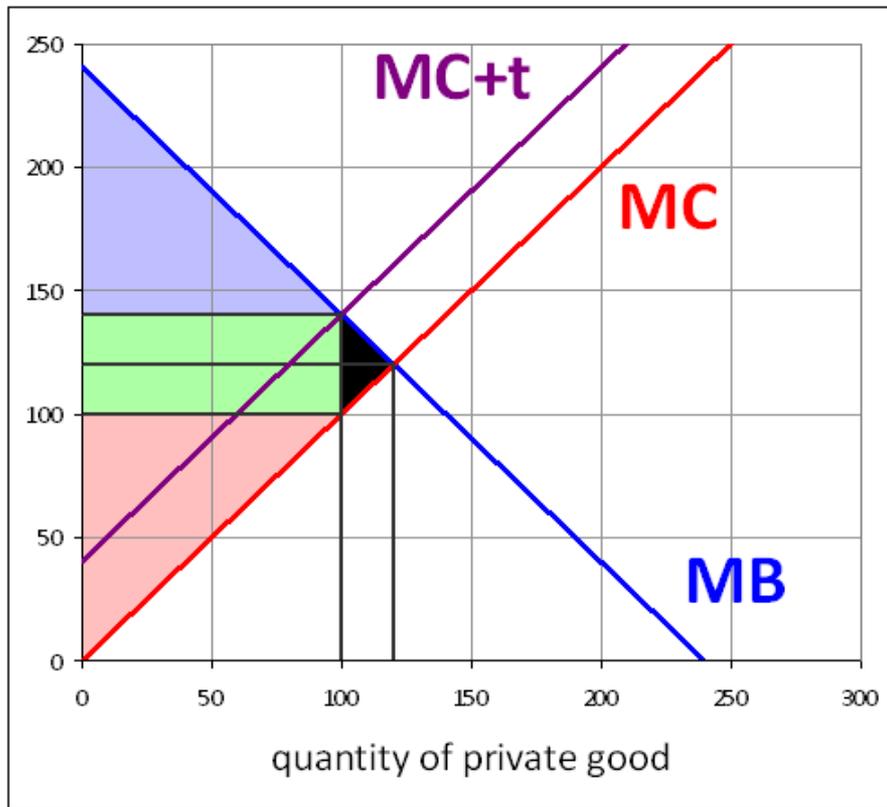
Since **MC = 100**, the tax revenue of **\$4000** can buy 40 units of the public good.

The **orange** area represents the tax revenue spent. The **orange** area and the **yellow** area together represent consumer surplus, so the **yellow** area by itself is the net surplus gain from government action.



This net surplus gain is  $(40)(50+30)/2 = 1600$ .

# TWO-MARKET MODEL OF GOVERNMENT



$$MB = 240 - x$$

$$MC = x$$

$$t = 40$$

$$G = 4000$$

$$DWL = 400$$

$$CS = 5000$$

$$PS = 5000$$

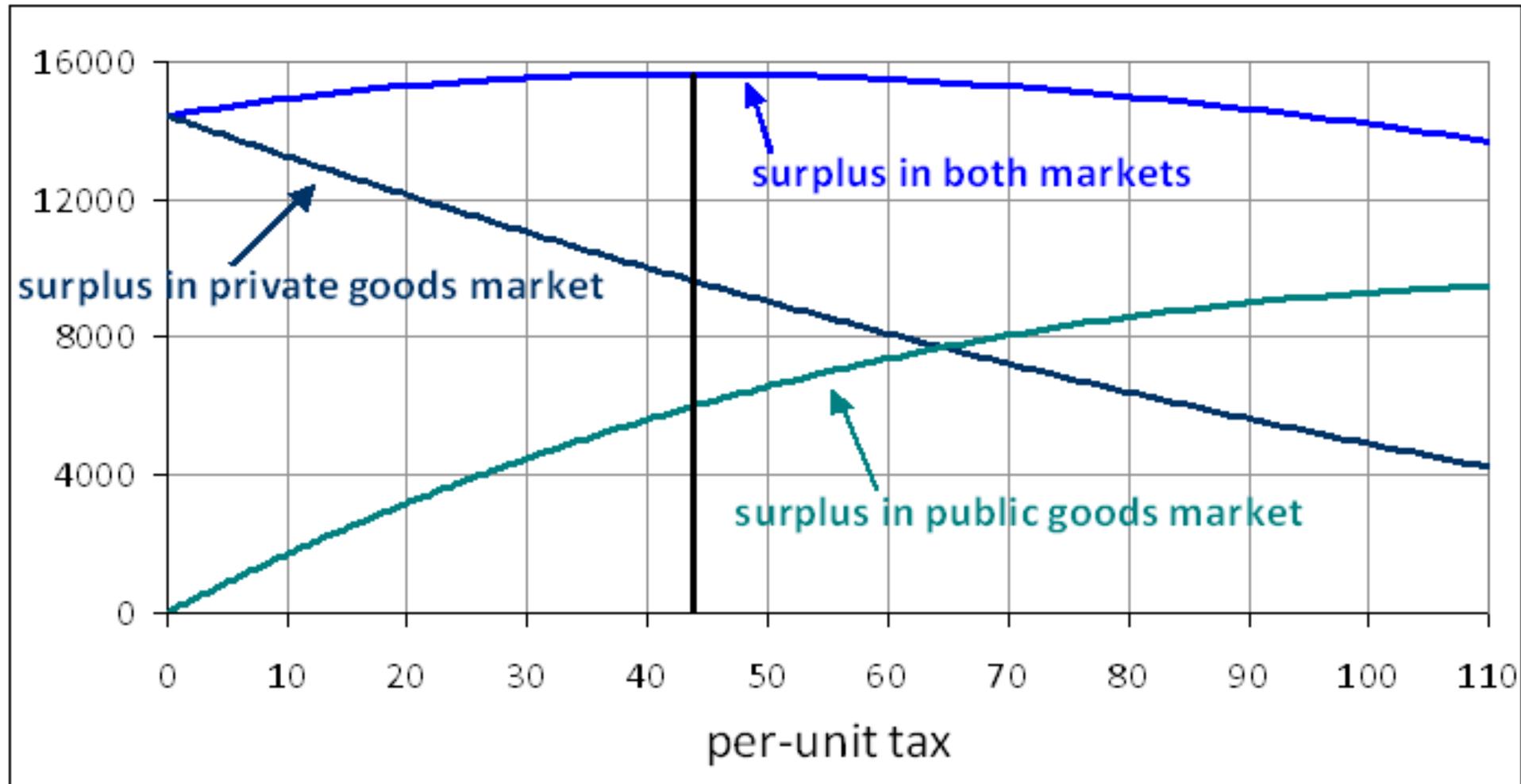
$$TES = 4000 + 5000 + 5000 = 14000 \text{ (down from 14400)}$$

**4000** spent;

surplus gained: **1600**

Compare to surplus loss of **400** from taxation of the private good: taxing and spending has improved total surplus by **1200**.

# OPTIMAL TAX AND SPENDING AMOUNT



**The optimal tax in this example is  $\approx 43.68$ , which generates a revenue of  $\approx 4,287$ , and increases total surplus by  $\approx 1,207$  (from 14,400 to  $\approx 15,607$ ).**

# **SUMMARY**

**Increasing the size of government can have both positive and negative effects.**

**On one hand, imposing taxes tends to cause deadweight loss in the taxed market.**

**On the other hand, using tax revenue to finance provision of public goods can increase total economic surplus.**