

**Problem set 4, due Tuesday 11/5/13**

**1. Household production and labor supply.** Ariel and Alana are sisters, with identical preferences over consumption and household time, but different productivities in the paid labor force. Let  $h$  be the share of time spent at home, and let  $c$  be consumption. Assume a simple budget constraint of  $c = (1 - h)w + k$ , where  $w$  represents a wage rate and  $k$  represents non-labor income.

For parts a-c, consider Ariel and Alana when they are young women with small children of their own. At this time, the preferences of each sister can be represented by the utility function  $U = h^3c$ .

**a)** Find the functions  $h(w, k)$  and  $c(w, k)$ , which give each sister's optimal choice of home time and consumption, as they depend on her respective wage rate and non-labor income.

**b)** If Ariel has a wage rate of  $w = 100$  and non-labor income of  $k = 20$ , find her optimal choice of hours and consumption.

**c)** If Alana has a wage rate of  $w = 30$  and non-labor income of  $k = 20$ , find her optimal choice of hours and consumption.

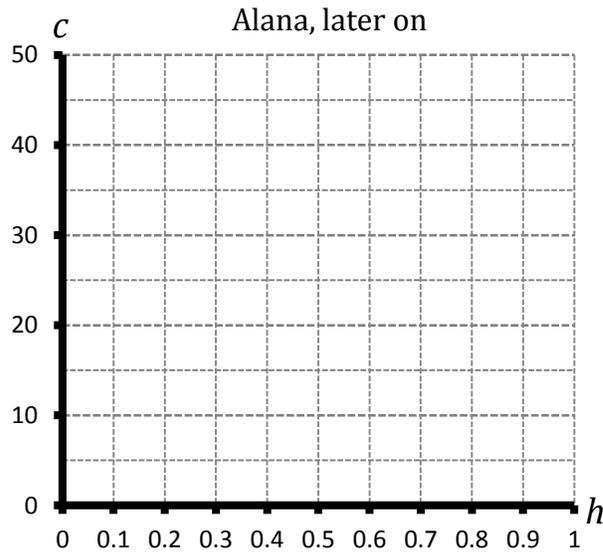
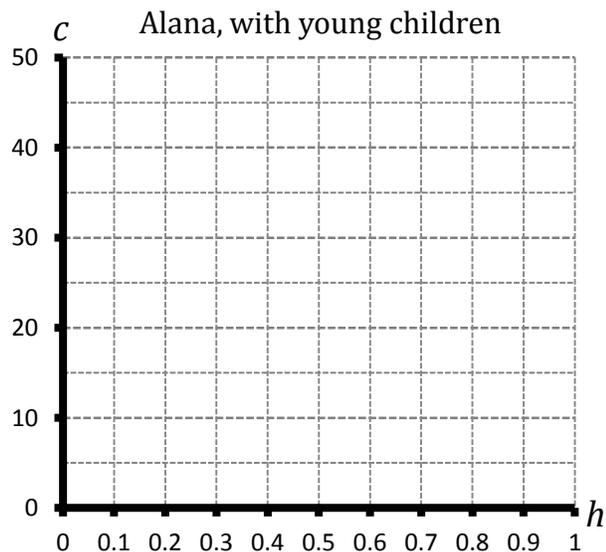
For parts d-f, consider Ariel and Alana when they are a bit older, and their children have grown up. At this time, the preferences of each sister can be represented by the utility function  $U = hc$ .

**d)** Find the functions  $h(w, k)$  and  $c(w, k)$ , which give each sister's optimal choice of home time and consumption, as they depend on her respective wage rate and non-labor income.

e) If Ariel has a wage rate of  $w = 100$  and non-labor income of  $k = 20$ , find her optimal choice of hours and consumption.

f) If Alana has a wage rate of  $w = 30$  and non-labor income of  $k = 20$ , find her optimal choice of hours and consumption.

g) Using the blank graphs below, draw Alana's budget constraint, her optimal combination of household time and consumption, her indifference curve when she's at this optimal combination, a higher indifference curve, and a lower indifference curve. Do this for each of her distinct life stages.



h) Using the four examples above (two sisters, each observed at two times), comment on the way in which the supply of paid labor depends on both the urgency of household work and the wage rate. Explain the intuition behind this.

**2) The life cycle and labor supply.** Suppose that Carl's life can be divided into two distinct time periods, period 1 and period 2. Defining  $e_1$  as leisure in period 1,  $e_2$  as leisure in period 2, and  $c$  as total lifetime consumption, Carl's preferences can be represented by the utility function  $U = e_1 e_2 c$ . Suppose a simple budget constraint of  $c = (1 - e_1)w_1 + (1 - e_2)w_2 + k$ , where  $w_1$  is Carl's wage rate in period 1,  $w_2$  is Carl's wage rate in period 2, and  $k$  is Carl's non-labor income.

**a)** Find the functions  $c(w_1, w_2, k)$ ,  $e_1(w_1, w_2, k)$ , and  $e_2(w_1, w_2, k)$ , which give Carl's optimal consumption, period 1 leisure, and period 2 leisure, as they depend on both wage rates and non-labor income.

**b)** Find Carl's optimal  $c$ ,  $e_1$ , and  $e_2$  if  $w_1 = 100$ ,  $w_2 = 80$ , and  $k = 0$ .

**c)** Find Carl's optimal  $c$ ,  $e_1$ , and  $e_2$  if  $w_1 = 130$ ,  $w_2 = 80$ , and  $k = 0$ .

**d)** Comment on the effect of a period 1 wage increase on optimal lifetime consumption, and optimal leisure time in both periods.

**e)** Find Carl's optimal  $c$ ,  $e_1$ , and  $e_2$  if  $w_1 = 100$ ,  $w_2 = 20$ , and  $k = 0$ .

**3) Comparative advantage.** Pat and Chris are partners. In a given hour, Pat can either make \$24 at work, or do 4 chores around the house. Likewise, Chris can either make \$12 at work, or do 3 chores around the house.

a) Who has an absolute advantage in what?

b) Who has a comparative advantage in what?

c) What trades of money for household chores might be mutually beneficial for Pat and Chris?