James Green-Armytage, econ 229, fall 2015

Supplemental problems for the midterm

1. Birthdays

a) 4 random people are in a room. Calculate the probability that at least two of them were born in the same month.

b) $n \in [1, 12]$ random people are in a room. Write an expression in terms of n that gives the probability that at least two people in the room were born in the same month.

2. Binomial distribution

a) Derive the PMF of the binomial distribution.

b) Describe the PMF of the binomial distribution in words. That is, what does it depend on, and what does its value tell us? Give a real life example of something that can be said to approximately follow a binomial distribution.

3. Dice experiment

a) Sketch the PMF of the value of a die, the sum of two dice, and the sum of ten dice.

b) Describe the distribution of the sum of 100 dice, in terms of mean, variance, and distribution type. (If you don't know the formula for the variance of a die value, you can represent it as σ^2.)

4. Distributions of random variables. For each of the following distributions (a) through (d), do each of the following: **(i)** Give a clearly explained example of something that can be said to approximately follow the distribution. **(ii)** Define the distribution's parameter(s) in words. **(iii)** Sketch the PDF or PMF, with your choice of parameter value(s), which you should state. Label both axes clearly.

a) Geometric

b) Poisson

c) Discrete uniform

d) Continuous uniform

5. Exponential distribution experiment

a) Sketch the PDF of the value of a draw from the exponential distribution, the sum of two draws from the exponential distribution, and the sum of 100 draws from the exponential distribution.

b) Describe the distribution of the sum of 100 draws from the exponential distribution, in terms of mean, variance, and distribution type. (If you don't know the formula for the mean or the variance of an exponential random variable, you can represent these as μ and σ^2.)

6. Central limit theorem

Write an informal but clear explanation of the central limit theorem. What does the theorem say? Give an example of how it is relevant in practical situations.

7. Confidence intervals, derivation

a) If $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is the sample mean of n draws from some distribution of a random variable x, with mean μ, variance σ^2, and standard deviation σ, derive formulae for $\mathrm{E}(\bar{x})$, $\mathrm{var}(\bar{x})$, and $\mathrm{sd}(\bar{x})$.

b) Defining $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ as the sample variance, and z^* as a value such that a standard normal random variable falls in the interval $[-z^*, z^*]$ with probability k, use your findings from (a) to write the formula for an interval where we believe the value μ should be located with probability k.

c) If $\bar{x}_A - \bar{x}_B =$ is the difference between your sample means from distribution A (with mean μ_A and variance σ_A^2) and distribution B (with mean μ_B and variance σ_B^2), derive formulae for $\mathrm{E}(\bar{x}_A - \bar{x}_B)$, $\mathrm{var}(\bar{x}_A - \bar{x}_B)$, and $\mathrm{sd}(\bar{x}_A - \bar{x}_B)$.

d) Defining $s_A^2 = \frac{1}{n_A-1}\sum_{i=1}^{n_A}(x_{Ai} - \bar{x}_A)^2$, $s_B^2 = \frac{1}{n_B-1}\sum_{j=1}^{n_B}(x_{Bi} - \bar{x}_B)^2$, and defining z^* as in part (b), use your findings from part (c) to write the formula for an interval where we believe the value $\mu_A - \mu_B$ should be located with probability k.

8. Confidence intervals, applied. I work for a cutting-edge drug company that has just started human trials for its new awesomeness potion. From our initial group of 100 human subjects, we've randomly assigned 60 to be in the experimental group (which is to receive a drink of the true awesomeness potion) and 40 to be in the control group (which is to drink only a placebo potion). One fortnight later, we measure the awesomeness level of each subject. (We have psychic ninjas who do this, in a double-blind setting, but let's not get into all of that right now.) In the end, we find that in the experimental group, the average awesomeness level is $\bar{x}_E = 55$, and the sample variance of awesomeness is $s_E^2 = 120$. We also find that in the control group, the average awesomeness level is $\bar{x}_C = 50$, and the sample variance of awesomeness is $s_C^2 = 110$.

a) Create a point estimate for the effect of the awesomeness potion on human awesomeness.

b) Construct 90%, 95%, and 99% confidence intervals around your point estimate.

One more note for the midterm

Don't forget to bring a calculator! Please bring either a scientific calculator or, failing that, an even more basic one. Graphing calculators are discouraged, and cell phones as calculators are not allowed. Graphing calculators are alienating and weird, but if you buy a humble scientific calculator and treat it okay, it could be your neat little buddy for years to come.