MIDTERM EXAM. ECON 229, FALL 2015. NAME: ________________________________

Fill in the blanks, and answer in the spaces provided. Show your work. Box your final answers.

1. Central limit theorem. In brief, what does the central limit theorem say? Why is it useful when estimating confidence intervals?

2. Cards. A standard deck of cards has 13 spades, 13 hearts, 13 diamonds, and 13 clubs. I draw four cards randomly from a standard deck. Write an expression for the probability that all four cards are of the same suit. (Your expression can use combinatorial notation.)

3. Marbles in an urn. I have an urn which contains two red marbles, two green marbles, and two blue marbles. I will draw three marbles from the urn *without replacement*. Find the probability that I will draw one marble of each color.

4. Bayesian inference. 1/5 of the population has a disease. Those with the disease will test positive 3/4 of the time. Those without the disease will test positive 1/4 of the time. If you have tested positive, what is the probability that you have the disease?

5. Dice experiment. Sketch the PMF (probability mass function) of the value of a die, the sum of two dice, and the sum of fifteen dice.

6. Exponential distribution experiment. Sketch the PDF (probability density function) of the value of a draw from an exponential distribution, the sum of two draws from an exponential distribution, and the sum of fifty draws from an exponential distribution.

7. Binomial distribution, part 1. If I flip a *fair* coin three times, what is the probability that I will get exactly two heads? Derive your answer with a probability tree, and without the use of the general formula for the binomial PMF.

8. Binomial distribution, part 2. If I flip a *fair* coin n times, what is the probability that I will get exactly k heads? Try to explain your answer as well as you can.

9. Sample average: Expected value and variance. Let $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ be the average of n random draws that I take from some random variable x, with mean and variance μ and σ^2. Derive $\mathrm{E}(\bar{x})$ and $\mathrm{var}(\bar{x})$, the expected value and variance of my sample average.

10. Sample average: Confidence interval. Defining $z^* = \Phi^{-1}\left(\frac{k+1}{2}\right)$, use your answer from the previous problem to construct the formula for an interval where we believe that μ should be located with probability k.

11. Difference between sample averages: Expected value and variance. Let $\bar{x}_A - \bar{x}_B = \left(\frac{1}{n_A}\sum_{i=1}^{n_A} x_{Ai}\right) - \left(\frac{1}{n_B}\sum_{j=1}^{n_B} x_{Bi}\right)$ be the difference between two sample averages, from populations A and B. Derive $\mathrm{E}(\bar{x}_A - \bar{x}_B)$ and $\mathrm{var}(\bar{x}_A - \bar{x}_B)$, the expected value and variance of this difference.

12. Difference between sample averages: Application. In a randomized trial, $n_E = 50$ subjects took a hair growth pill, and $n_C = 50$ subjects took a placebo pill. The hair length of both groups was measured one year later. The experimental group had average hair growth of $\bar{x}_E = 12$ inches, with a sample variance of $s_E^2 = 16$. The control group had average hair growth of $\bar{x}_C = 10$ inches, with a sample variance of $s_C^2 = 16$. Estimate the effect of the hair growth pill; that is, give both a point estimate for $\mu_E - \mu_C$ and a 95% confidence interval around your point estimate. You can use a formula here if you like.