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Probabilities of Standard and Invented Poker Hands

Here we find the number of unique five-card combinations that meet the criteria for the eight standard poker hands, and five additional invented hands. Since each unique combination is equally probable, the probability of being dealt each type of hand can be calculated as the number of combinations meeting the criteria, divided by the total number of unique five-card combinations, which is $\binom{52}{5} = 2{,}598{,}960$.

For each type of hand, we calculate the number of combinations meeting the criteria as the product of the number of possible answers to a series of questions, which are stated in the same order as the components of the calculations.

1. Standard Hands

1.1. Straight Flush (including royal flush)

Which numbers are included? Which suit do they share?

$$10 \cdot 4 = 40$$

1.2. Four of a Kind

Which number is quadrupled? What is the fifth card?

$$13 \cdot 48 = 624$$

1.3. Full House

Which number is tripled? Which number is doubled? Which suits does the tripled number have? Which suits does the doubled number have?

$$13 \cdot 12 \cdot \binom{4}{3} \cdot \binom{4}{2} = 3{,}744$$

1.4. Flush (but not a straight flush)

Which do the cards share? Which numbers from the suit are included? (Then subtract the frequency from 1.1.)

$$4 \cdot \binom{13}{5} - 40 = 5{,}108$$

1.5. Straight (but not a straight flush)

Which numbers are included? Which suit does each number have? (Then subtract the frequency from 1.1.)

$$10 \cdot 4^5 - 40 = 10{,}200$$

1.6. Three of a Kind (but not a full house or a four of a kind)

Which number is tripled? Which suits does the tripled number have? What are the numbers of the other two cards? What are the suits of the other two cards?

$$13 \cdot \binom{4}{3} \cdot \binom{12}{2} \cdot 4^2 = 54{,}912$$

1.7. Two Pair (but not a full house)

Which numbers are doubled? Which suits does each of the doubled numbers have? What is the other card?

$$\binom{13}{2} \cdot \binom{4}{2}^2 \cdot 44 = 123{,}552$$

1.8. One Pair

Which number is doubled? Which suits does the doubled number have? What are the numbers of the three other cards? What are the suits of the three other cards?

$$13 \cdot \binom{4}{2} \cdot \binom{12}{3} \cdot 4^3 = 1{,}098{,}240$$

2. Invented Hands

2.1. Faces

All five cards are face cards; i.e. any mixture of jacks, queens, and kings; suit not important.

Which face cards are included?

$$\binom{12}{5} = 792$$

2.2. Skipping Straight

Sequences of cards that skip intervals of one each time, i.e. $\{A, 3, 5, 7, 9\}$ *or* $\{2, 4, 6, 8, 10\}$*. Suit not important. Face cards don't work for this hand.*

Which numbers are in the straight? Which suits are each number?

$$2 \cdot 4^5 = 2{,}048$$

2.3. Court

One king, one queen, one jack, and two numbered cards (A through 10).

Which king, which queen, and which jack? Which two numbered cards?

$$4^3 \cdot \binom{40}{2} = 49{,}920$$

2.4. Rainbow

Four numbered cards (A through 10) that represent each suit exactly once, plus a face card (jack, queen, or king) of any suit.

Which spade, which heart, which diamond, and which club? Which face card?

$$10^4 \cdot 12 = 120{,}000$$

2.5. Monochrome

Either all cards are black (spades and clubs), or all cards are red (hearts and diamonds).

Which color (black or red)? Which cards from that color?

$$2 \cdot \binom{26}{5} = 131{,}560$$