MIDTERM EXAM, ECON 201, FALL 2014   NAME: ______________________________

Answer in the space provided. You must show correct work for full credit.

1. Derive Cobb-Douglas demand function. Suppose that a consumer's utility is given by $U = x_1^{\alpha} x_2^{\beta}$, where x_1 and x_2 represent the quantities of the two goods he consumes. Let p_1 and p_2 be the prices of the two goods, and let m be the consumer's income. Assume that α, β, p_1, p_2, and m are positive numbers. Here we can assume also that $\alpha + \beta = 1$.

a) Write the equation for the consumer's budget line.

b) Suppose that we create a graph with x_2 on the vertical axis and x_1 is on the horizontal axis. Find the slope of the budget line on this graph.

c) Given the same graph, find the slope of the consumer's indifference curve.

d) Using your answers to b and c, write an equation that indicates that the consumer is choosing a utility-maximizing bundle.

e) Now we want to solve for the consumer's demand functions. Identify the variables we are solving for, and the equations we can use.

f) Use algebra to find the consumer's demand functions for goods 1 and 2.

2. Derive perfect substitutes demand function. Suppose that a consumer's utility is given by $U = \alpha x_1 + \beta x_2$. As before, suppose that p_1, p_2, m, α, and β are positive numbers.

a) Write an inequality in terms of α, β, p_1, and p_2 which implies that the consumer should only consume good 1.

b) If this inequality holds, how much of good 1 should the consumer buy?

c) Fill in the demand functions below. You don't have to worry about the case in which the consumer is indifferent among all affordable bundles.

$$x_1(p_1, p_2, m) = \begin{cases} \qquad\qquad\qquad \text{if} \quad > \\ \\ \qquad\qquad\qquad \text{otherwise} \end{cases} \qquad x_2(p_1, p_2, m) = \begin{cases} \qquad\qquad\qquad \text{if} \quad > \\ \\ \qquad\qquad\qquad \text{otherwise} \end{cases}$$

3. Short answers. (Please be as clear and precise as possible.)

a) Give three definitions of a 'marginal rate of substitution'.

b) Why is it possible to perform a monotonic transofmation on a utility function without changing the demand function that it imples?

4. Analysis of demand. Suppose that Ariel spends all of her money on fancy pasta (good 1) and beans (good 2), with prices p_1 and p_2. Ariel's utility function is given by $U = 10\sqrt{x_1} + x_2$, where x_1 and x_2 are the quantities of fancy pasta and beans that she consumes, respectively. Ariel has an income of m. Assume that m is large enough to avoid a 'corner solution' (where her optimal consumption of beans is zero.

a) Find Ariel's demand function for fancy pasta, $x_1(p_1, p_2, m)$.

b) Find Ariel's demand function for beans, $x_2(p_1, p_2, m)$.

c) In terms of p_1 and p_2, how large does m have to be to avoid a corner solution?

d) Are fancy pasta and beans complements? Substitutes? Explain clearly how you know.

e) Assuming that we don't have a corner solution, is fancy pasta (good 1) a normal good, or an inferior good? How about beans (good 2)?

f) Is fancy pasta (good 1) a Giffen good? Explain clearly how you know.

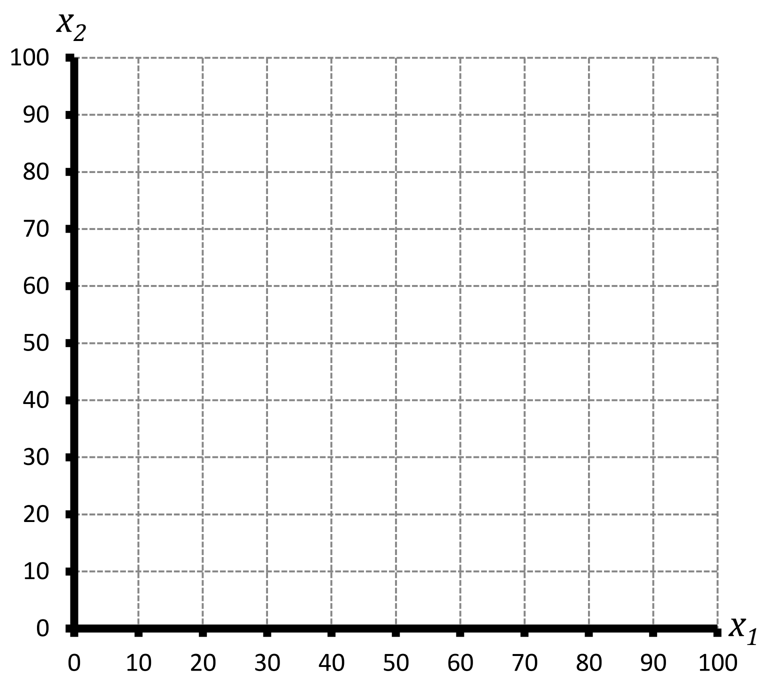
g) Find the price elasticity of Ariel's demand for fancy pasta (good 1).

5. Graphing. Suppose that my utility function is given by $U = \min\left\{\frac{1}{2}x_1, x_2\right\}$. Here, x_1 and x_2 represent fluid ounces of whiskey and sweet vermouth, respectively, and my utility is given by the number of Manhattans I can make this week, assuming that I need a strict 2:1 ratio, and assuming free availability of ice, maraschino cherries, etc. The price per ounce of whiskey is $p_1 = \$1/10$, the price per ounce of vermouth is $p_2 = \$1/10$, and I have $6 to spend on cocktails this week.

a) On the graph below, draw my budget line. Label this BL.

b) Draw an indifference curve indicating the bundles that give me utilities of 10, 20, 30, and 40, labeling them $U = 10$, $U = 20$, etc.

c) How much of each good should I buy, in order to maximize my utility? Circle this bundle on the graph, and label it 'optimal bundle'.



d) Now let's generalize to any values of p_1, p_2, and m, while keeping my utility function as is. Write the equations for the budget line and the line that passes through all of my indifference curve kinks, and use these equations to solve for the demand function $x_1(p_1, p_2, m)$.

6. Substitution effects and income effects. Suppose that there is a lady named Daisy who likes peanut butter and cheese. Given that x_1 is the quantity of peanut butter she consumes per day, and x_2 is the amount of cheese she consumes per day, her preferences can be represented by the utility function $U = x_1 {x_2}^2$.

a) Initially, Daisy faces prices $p_1 = 1$ and $p_2 = 1$, and has daily income $m = 90$. Find x_1 and x_2 (her demand for peanut butter and cheese) given these prices and this income.

b) Suppose that the price of peanut butter changes to $p_1' = 2$. In order to make Daisy's original bundle (from part a) just barely affordable given this price change, you must also change her income to m'. What is m'?

c) If the price of peanut butter is $p_1' = 2$, and Daisy's income is m' (from part b), find x_1 and x_2.

d) Now, suppose that the price of peanut butter changes to $p_1' = 2$, but Daisy's income remains at its initial value of $m = 90$. Find x_1 and x_2 in this case.

e) We have analyzed Daisy's response to an increase in the price of peanut butter from $p_1 = 1$ to $p_1' = 2$. In the table below, indicate the substitution effect, the income effect, and the total effect of this price change on her demand for both goods. Use + and – signs to indicate whether the effect is positive or negative.

	peanut butter	cheese
substitution effect		
income effect		
total effect		