Second test, ECON 201, fall 2014. NAME: ________________________________

Answer in the space provided. Show correct work for full credit. Box your final answers.

1. Profit maximization with one input. Suppose that a (perfectly competitive) firm has the production function $y = f(x) = 12x^{1/3}$, where x and y are the firm's input and output quantities, respectively. Let p be the price of the firm's output, and let w be the price of the firm's input. Assume that the firm has no other costs.

a) Write the $\pi(x)$ function (profit as a function of the input quantity x) in the most explicit possible form, and use this function to solve for the profit-maximizing input quantity, x^*, in terms of p and w.

b) Write the $\pi(y)$ function (profit as a function of the output quantity y) in the most explicit possible form, and use this function for the profit-maximizing output quantity, y^*, in terms of p and w.

c) Use either or both of your calculations above to find the values of R (revenue), C (cost), and π (profit) when the firm is choosing its input and output in a profit-maximizing manner. Again, each of these depend only on p and w.

2. Profit maximization with two inputs. Suppose that a (perfectly competitive) firm has the production function $y = f(x_1, x_2) = {x_1}^{1/5}{x_2}^{1/5}$, where y is its output quantity, and x_1 and x_2 are the quantities it uses of inputs 1 and 2, respectively. Let $p = 10{,}000$ be the output price, and let $w_1 = 1$ and $w_2 = 4$ be the input prices.

a) Your goal is to find the profit-maximizing values of the input and output quantities: x_1^*, x_2^*, and y^*. There are a few possible ways to do this, and you may choose which one you prefer. In the space below, explain clearly how you plan to proceed, providing clear justification for your approach.

b) You have explained your method above; below, you will implement it. Find the numerical values of x_1^*, x_2^*, and y^*.

3. Cost function analysis. Suppose that each firm in a (perfectly competitive) industry has the same cost function. $C(y) = \frac{1}{10}y^2 + 4y + 10$, where y is the firm's output quantity.

a) Find a firm's average variable cost function, $AVC(y)$.

b) Find a firm's average cost function, $AC(y)$.

c) Find a firm's marginal cost function, $MC(y)$.

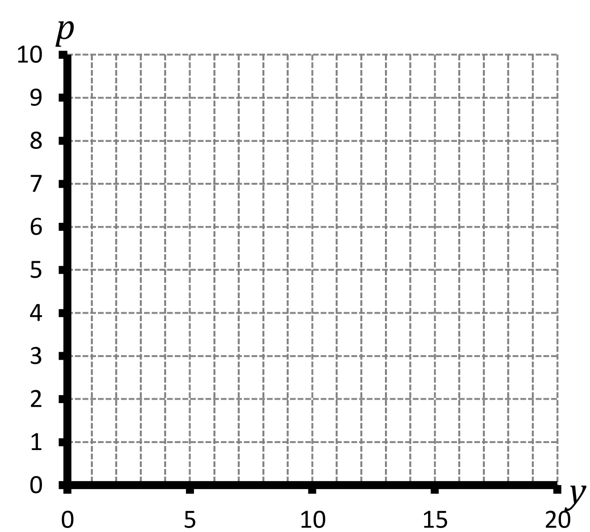
d) What value of y minimizes a firm's average cost?

e) What is the minimum value of a firm's average cost?

f) In the short run (where each firm is committed to paying its fixed cost), what is a firm's supply function, $y(p)$?

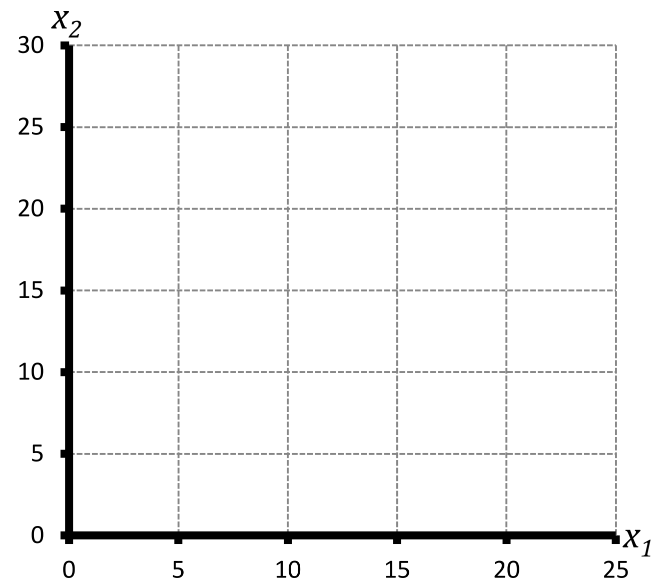
g) Now consider the long run, in which firms may enter and exit. Suppose that market supply is given by $S(p) = ny(p)$, where n is the number of firms, and that market demand is given by $D(p) = 198 - 8p$. Find the equilibrium number of firms, n^*.

h) On the graph to the right, draw an individual firm's marginal cost (MC), average cost (AC), and average variable cost (AVC) functions.



4. Cost minimization. Suppose that $y = f(x_1, x_2) = \min\{2x_1, x_2\}$ is the firm's production function, where x_1 and x_2 are input quantities. Let the input prices be $w_1 = 2$ and $w_2 = 1$.

a) On the graph to the right, draw the isoquant for $y = 10$. Draw isocost lines for $C = 5$, $C = 10$, $C = 15$, $C = 20$, $C = 25$, and $C = 30$. What is the minimum cost at which 10 units of output can be produced? __________



b) Still assuming the same production function and input prices, find the cost function, $C(y)$.

5. Definitions. Define each of the following terms as clearly as possible:

a) Marginal product.

b) Isoquant.

c) Decreasing returns to scale.

6. Cost function from production function, example 1. Assume perfect competition. Let x be the firm's input quantity, let y be the output quantity, let $y = f(x) = 4\sqrt{x}$ be the production function, let w be the input price, and let the purchase of this input be the firm's only cost.

a) Find $x(y)$, i.e. the input quantity needed to produce an output quantity of y.

b) Find $C(y)$, i.e. the cost needed to produce an output quantity of y.

c) Find $MC(y)$, i.e. the marginal cost of production, given an output quantity of y.

7. Cost function from production function, example 2. Suppose that everything is as in the previous problem except that the firm's production function is $y = f(x) = 4x^2$.

a) Find $x(y)$, i.e. the input quantity needed to produce an output quantity of y.

b) Find $C(y)$, i.e. the cost needed to produce an output quantity of y.

c) Find $MC(y)$, i.e. the marginal cost of production, given an output quantity of y.

8. Cost function from production function, reflection.

a) Using examples 1 and 2, comment on the relationship between the production function's returns to scale, and whether marginal cost increases or decreases with output quantity. Be as clear as possible about why this follows.

b) Which of these examples is more compatible with perfect competition. Explain clearly.