Third test, ECON 201, fall 2014. NAME: ___________________________________

Answer in the space provided. Show correct work for full credit. Box your final answers.

1. Intertemporal choice. Suppose that I have the utility function $U = 3\ln c_1 + 2\ln c_2$, where c_1 is my consumption in year 1 and c_2 is my consumption in year 2. In each year I earn 600; i.e. $m_1 = 600$ and $m_2 = 600$. I can either save or borrow; in either case the interest rate is $1/5$, or 20%.

Find the value s^* that maximizes my utility. If s^* is positive, it represents how much I should save in year 1, and if s^* is negative, it represents how much I should borrow in year 1. Find the consumption values c_1^* and c_2^* that result from my optimal saving or borrowing amount, s^*.

2. Group-based price discrimination. I write poems and sell them in Woodstock, NY, which is populated entirely by artists and bankers. Each artist has the inverse demand curve $p^A(q^A) = 18 - \frac{1}{10}q^A$, and each banker has the inverse demand curve $p^B(q^B) = 12 - \frac{1}{10}q^B$, where p and q represent the price and quantity of my poems. There are *twice as many* bankers as artists. I can supply poems with zero marginal cost.

Suppose for the sake of this problem that I charge my customers by the poem, and that I can charge different prices to artists and bankers. Find the profit-maximizing prices and quantities for each type of consumer. Also, find my total profit from a representative group of three consumers, i.e. $\pi = \pi^A + 2\pi^B$.

$q^A =$ ____ $p^A =$ ____ $q^B =$ ____ $p^B =$ ____ $\pi =$ ____

3. Quantity-based price discrimination. I am still selling poems in Woodstock. My marginal cost is still zero, and my market still consists of artists who have inverse demand $p^A(q^A) = 18 - \frac{1}{10}q^A$ and bankers who are twice as numerous and who have inverse demand $p^B(q^B) = 12 - \frac{1}{10}q^B$. However, two things have changed: First, the town council has prohibited me from price-discriminating by group. Second, I have decided to charge customers for packages of multiple poems rather than charging by the individual poem.

For parts a-c, suppose that I offer a 180-poem package and a 120-poem package, in exchange for fees F_{180} and F_{120}, respectively.

a) Find the profit-maximizing value of F_{120}.

b) Write the incentive-compatibility constraint on the price of the larger package, and use it to find the profit-maximizing value of F_{180}.

c) Find my profit from a representative group of one artist and two bankers, $\pi = F_{180} + 2F_{120}$.

In parts d-f, suppose that I offer a 180-poem package and an x-poem package, in exchange for fees F_{180} and F_x, respectively. Use the notation V_q^A and V_q^B to indicate the total value of a q-minute package to a type A person and a type B person, respectively.

d) Find an expression for profit $\pi = F_{180} + 2F_x$ in terms of V_{180}^A, V_x^A, and V_x^B.

e) Use the expression from part (d) to solve for the profit-maximizing value of x.

f) Given the value of x you found in part e, calculate numerical values for F_{180}, F_x, and π.

4. Exchange. Arnold has 24 xylophones and 12 yaks. Betty has 12 xylophones and 24 yaks. Arnold and Betty have the utility functions $U_A = {x_A}^2 y_A$, and $U_B = {x_B}^2 y_B$, respectively. (Here, x_A represents Arnold's consumption of xylophones, y_B represents Betty's consumption of yaks, and so on.) Arnold and Betty may trade xylophones for yaks, but the total number of each is fixed. Let $p \equiv p_x/p_y$ represent the ratio of the xylophone price and the yak price. Suppose that both Arnold and Betty take the value of p as given.

a) Write down two equations in terms of x_A, y_A, and p which can be used to find Arnold's utility-maximizing quantities of xylophones and yaks when trading at a price ratio of p. Use these to solve for x_A in terms of p.

b) Repeat part (a) for Betty, solving for x_B in terms of p.

c) Use your answers from parts (a) and (b) to solve for the value of p at which demand and supply are equal.

d) Use your answer from part (c) to find the numerical values of x_A, y_A, x_B, and y_B in the competitive equilibrium.

5. Perfect competition and monopoly. Suppose that the marginal cost of producing a particular good is 3, and that there are no other costs. The market price is given by the inverse demand function $p(Y) = 15 - \frac{1}{10}Y$, where Y is the total quantity of output.

a) If this market is perfectly competitive, find the price (p^*) and quantity (Y^*) in equilibrium.

b) If there is only one seller, who is a profit maximizing monopolist, find the equilibrium price and quantity.

6. Duopoly. Assume the same marginal cost and inverse demand function as above, but now suppose that there are two firms who produce this good; firm 1 chooses an output quantity y_1, firm 2 chooses an output quantity y_2, and the resulting total output quantity is $Y = y_1 + y_2$.

a) Find the reaction functions $y_1^r(y_2)$ and $y_2^r(y_1)$, which give the profit-maximizing quantity of each firm, dependent on the other firm's quantity.

b) Find the Cournot equilibrium values of y_1^*, y_2^*, and p^*.

c) Find the Stackelberg equilibrium values of y_1^*, y_2^*, and p^*, assuming that firm 1 is the leader (decides its quantity first) and firm 2 is the follower (decides its quantity second).