1. Quantity-based price discrimination. I have achieved a monopoly on the cellular service market in Red Hook, NY, which is populated entirely by astronauts and barbers. Each astronaut has the inverse demand curve $p^A(q^A) = 8 - \frac{1}{10}q^A$, and each barber has the inverse demand curve $p^B(q^B) = 4 - \frac{1}{10}q^B$, where p and q represent the price and quantity of cell phone minutes per day. There are *twice as many* barbers as astronauts. I cannot tell the difference between astronauts and barbers. I can supply phone minutes with zero marginal cost.

For parts a-c, suppose that I offer an 80-minute package and a 40-minute package, in exchange for fees F_{80} and F_{40}, respectively

a) Find the profit-maximizing value of F_{40}.

b) Write the incentive-compatibility constraint on the price of the larger package, and use it to find the profit-maximizing value of F_{80}.

c) Find my profit from a representative group of one astronaut and two barbers, $\pi = F_{80} + 2F_{40}$.

In parts d-f, suppose that I offer an 80-minute package and an x-minute package, in exchange for fees F_{80} and F_x, respectively. Use the notation V_q^A and V_q^B to indicate the total value of a q-minute package to a type A person and a type B person, respectively.

d) Find an expression for profit $\pi = F_{80} + 2F_x$ in terms of V_{80}^A, V_x^A, and V_x^B.

e) Use the expression from part (d) to solve for the profit-maximizing value of x.

f) Given the value of x you found in part e, calculate numerical values for F_{80}, F_x, and π.

2. Group-based price discrimination. Suppose as in problem 1 that I am selling cell phone minutes (which cost me nothing) to astronauts with inverse demand $p^A(q^A) = 8 - \frac{1}{10}q^A$ and barbers with inverse demand $p^B(q^B) = 4 - \frac{1}{10}q^B$. But now, for whatever reason, I am not allowed to structure my prices in terms of fees for packages containing a certain number of minutes; instead, I have to charge strictly by the minute.

a) First, suppose that I can distinguish between astronauts and barbers, and charge them separate prices. Find the profit-maximizing prices and quantities for each type of consumer. Also, find my total profit from a representative group of three consumers, i.e. $\pi = \pi^A + 2\pi^B$.

$q^A =$ ____ $\quad p^A =$ ____ $\quad q^B =$ ____ $\quad p^B =$ ____ $\quad \pi =$ ____

b) Second, suppose that I cannot charge separate prices to astronauts and barbers. Find the demand curve for a representative group of three consumers $Q(p) = q^A(p) + 2q^B(p)$. Use this to find my profit-maximizing price and quantity.

$Q =$ ____ $\quad p =$ ____

c) [Placeholder for something else.]

3. Exchange. Annie has 36 xylophones and 18 yaks. Bill has zero xylophones and 18 yaks. Annie and Bill have the utility functions $U_A = {x_A}^2 y_A$, and $U_B = x_B {y_B}^2$, respectively. (Here, x_A represents Annie's consumption of xylophones, y_B represents Bill's consumption of yaks, and so on.) Annie and Bill may trade xylophones for yaks, but the total number of each is fixed. Let $p \equiv p_x/p_y$ represent the ratio of the xylophone price and the yak price. Suppose that both Annie and Bill take the value of p as given.

a) Find an equation, with y_A on the left-hand side and some function of x_A on the right hand side, which represents all the Pareto efficient divisions of the two goods, i.e. those such that there can be no mutual gains from trade.

b) Write down two equations in terms of x_A, y_A, and p which can be used to find Annie's utility-maximizing quantities of xylophones and yaks when trading at a price ratio of p. Use these to solve for x_A in terms of p.

c) Repeat part (b) for Bill, solving for x_B in terms of p.

d) Use your answers from parts (b) and (c) to solve for the value of p at which demand and supply are equal.

e) Use your answer from part (d) to find the numerical values of x_A, y_A, x_B, and y_B in the competitive equilibrium.