

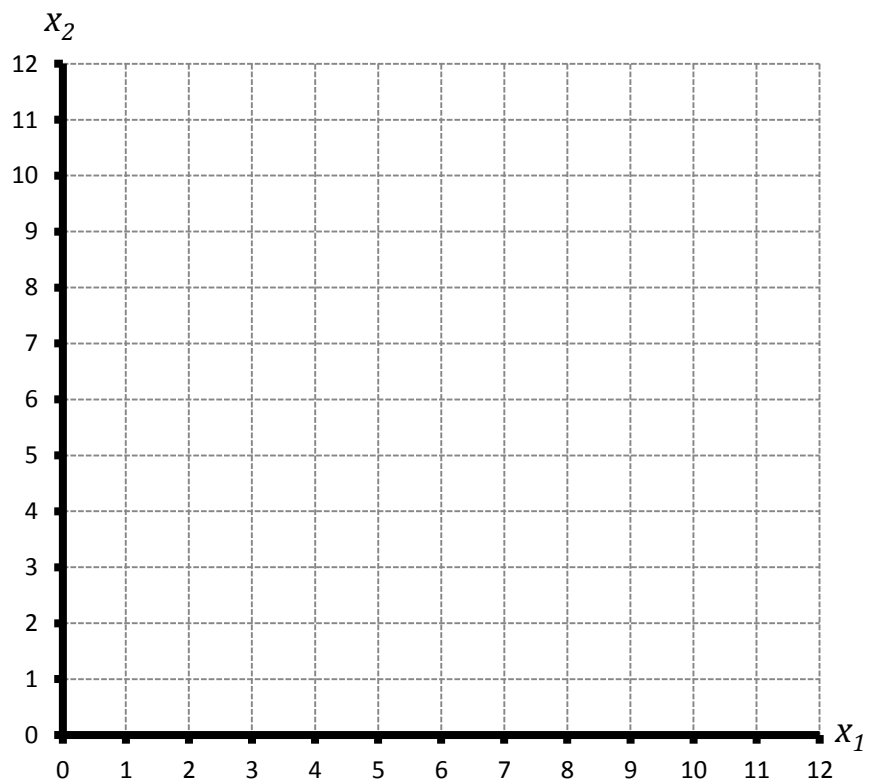
Answer in the space provided. You must show correct work for full credit.

**1. Graphing.** Suppose that a consumer's utility is given by  $U = x_1x_2$ , where  $x_1$  and  $x_2$  represent the quantities of the two goods he consumes. Suppose that price and income are  $p_1 = 3$ ,  $p_2 = 1$ , and  $m = 12$ .

a) Draw the budget line.

b) Draw the indifference curves that correspond to  $U = 6$ ,  $U = 12$ , and  $U = 24$ . You should be precise, and plot at least four exact points on each curve where both  $x_1$  and  $x_2$  are whole numbers.

c) Use calculus and algebra to find the utility-maximizing bundle, and mark this point on the graph. What are the two equations that must be satisfied at this point?



**2. Derive perfect substitutes demand functions.** Suppose that a consumer's utility is given by  $U = \alpha x_1 + \beta x_2$ . Assume that  $p_1, p_2, m, \alpha$ , and  $\beta$  are positive numbers.

a) Write an inequality in terms of  $\alpha, \beta, p_1$ , and  $p_2$  which implies that the consumer should only consume good 1.

b) If this inequality holds, how much of good 1 should the consumer buy?

c) Fill in the demand functions below. You don't have to worry about the case in which the consumer is indifferent among all affordable bundles.

$$x_1(p_1, p_2, m) = \begin{cases} \text{if } > \\ \text{otherwise} \end{cases} \quad x_2(p_1, p_2, m) = \begin{cases} \text{if } > \\ \text{otherwise} \end{cases}$$

**3. Derive perfect complements demand functions.** Suppose that a consumer's utility is given by  $U = \min\{\alpha x_1, \beta x_2\}$ . Assume that  $p_1, p_2, m, \alpha$ , and  $\beta$  are positive numbers.

a) Write and box two distinct equations that can be used to construct the demand functions.

b) Solve your two equations simultaneously to find the demand functions  $x_1(p_1, p_2, m)$  and  $x_2(p_1, p_2, m)$ .

c) Argue that the two goods are complements, using only the demand functions.

**4. Substitution effects and income effects.** There is a lady named Daisy who likes peanut butter and cheese. Given that  $x_1$  is the quantity of peanut butter she consumes per month, and  $x_2$  is the amount of cheese she consumes per month, her preferences can be represented by the utility function  $U = x_1^3 x_2$ .

**a)** Initially, Daisy faces prices  $p_1 = 2$  and  $p_2 = 2$ , and has monthly income  $m = 640$ . Find  $x_1$  and  $x_2$  (her demand for peanut butter and cheese) given these prices and this income.

**b)** Suppose that the price of peanut butter changes to  $p'_1 = 3$ . In order to make Daisy's original bundle (from part a) just barely affordable given this price change, you must also change her income to  $m'$ . What is  $m'$ ?

**c)** If the price of peanut butter is  $p'_1 = 3$ , and Daisy's income is  $m'$  (from part b), find  $x_1$  and  $x_2$ .

**d)** Now, suppose that the price of peanut butter changes to  $p'_1 = 3$ , but Daisy's income remains at its initial value of  $m = 640$ . Find  $x_1$  and  $x_2$  in this case.

**e)** We have analyzed Daisy's response to an increase in the price of peanut butter from  $p_1 = 2$  to  $p'_1 = 3$ . In the table below, indicate the substitution effect, the income effect, and the total effect of this price change on her demand for both goods. Use + and – signs to indicate whether each effect is positive or negative.

|                     | $x_1$ (peanut butter) | $x_2$ (cheese) |
|---------------------|-----------------------|----------------|
| substitution effect |                       |                |
| income effect       |                       |                |
| total effect        |                       |                |

**5. Analysis of demand.** For each of the following utility functions, answer the following two questions. First, are the goods normal, inferior, or neither? (If the two goods are different in this regard, specify this.) Second, are the goods complements, substitutes, or neither? You should answer both questions with a mixture of mathematical derivation and intelligible explanation in English; try to give as much intuition as possible.

a) **Cobb-Douglas** utility:  $U = x_1^\alpha x_2^\beta$ .

b) **Perfect substitutes** utility:  $U = \alpha x_1 + \beta x_2$ .

c) **Perfect complements** utility:  $U = \min\{\alpha x_1, \beta x_2\}$ .

d) (A specific type of) **quasilinear** utility:  $U = \alpha\sqrt{x_1} + x_2$ . (If you like, you can restrict your discussion to the case where  $m$  is large enough to avoid a corner solution.)