SECOND TEST. ECON 201, fall 2015.   NAME: ________________________________

Answer in the space provided. Show correct work for full credit. Box your final answers.

1. Profit maximization with one input. Let y and p be the quantity produced and price of a firm's output. Let x and w be the quantity and price of the firm's production input. Assume that the firm is a price-taker, with no other costs aside from this input.

a) Let $y = f(x)$ be the firm's production function. Assume that $f'(x) > 0$ and $f''(x) < 0$. Write an expression for profit (π) in terms of x, and use this to derive an equation that characterizes x^*, the profit-maximizing value of x. Be as explicit as possible throughout.

b) Let $C(y)$ be the firm's cost function. Assume that $C'(y) > 0$ and $C''(y) > 0$. Write an expression for profit (π) in terms of y, and use this to derive an equation that characterizes y^*, the profit maximizing value of y. Be as explicit as possible throughout.

For parts (c) – (e), suppose that $f(x) = 300x^{1/3}$, $p = 2$, and $w = 8$

c) Write $\pi(x)$ in the most explicit form, and use it to find x^*.

d) Write $\pi(y)$ in the most explicit form, and use it to find y^*.

e) Verify that your answers in parts (c) and (d) are consistent with each other.

2. Profit maximization with two inputs. Suppose that a (perfectly competitive) firm has the production function $y = f(x_1, x_2) = x_1{}^{2/5} x_2{}^{1/5}$, where y is its output quantity, and x_1 and x_2 are the quantities it uses of inputs 1 and 2, respectively. Let $p = 2{,}000$ be the output price, and let $w_1 = 1$ and $w_2 = 4$ be the input prices.

a) Find the profit-maximizing values of the input and output quantities: x_1^*, x_2^*, and y^*.

b) What type of returns to scale does this production function have? Is marginal cost $C'(y)$ decreasing, constant, or increasing in y? Explain as clearly as you can why your two findings here are intuitively consistent with each other.

3. Long run competitive equilibrium. Suppose that each firm in a perfectly competitive industry has the cost function $C(y) = \frac{1}{100}y^2 + y + 100$, where y is the firm's output quantity.

a) Find a firm's average variable cost function, $AVC(y)$.

b) Find a firm's average cost function, $AC(y)$.

c) Find a firm's marginal cost function, $MC(y)$.

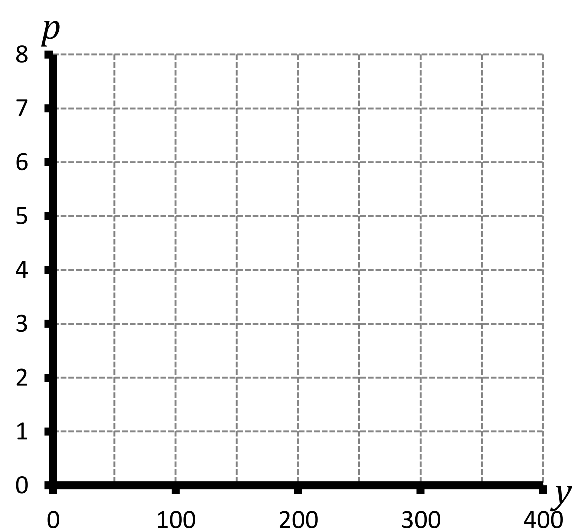
d) What value of y minimizes a firm's average cost?

e) What is the minimum value of a firm's average cost?

f) In the short run (where each firm is committed to paying its fixed cost), what is a firm's supply function, $y(p)$?

g) Now consider the long run, in which firms may enter and exit. Suppose that market supply is given by $S(p) = ny(p)$, where n is the number of firms, and that market demand is given by $D(p) = 2900 - 200p$. Find the equilibrium number of firms, n^*.

h) On the graph to the right, draw an individual firm's marginal cost (MC), average cost (AC), and average variable cost (AVC) functions.



4. Cost minimization: perfect complements. Suppose that a firm's production function is $y = f(x_1, x_2) = \min\{\alpha x_1, \beta x_2\}$, where x_1 and x_2 are input quantities, and $\alpha > 0$, $\beta > 0$ are constants. Let the input prices be w_1 and w_2.

a) Write expressions for the input quantities x_1 and x_2 that one would use to produce an output quantity of y in a cost-minimizing way.

b) Use your answer from part (a) to write a cost function, $C(y)$. Write this in a form that is as simple and as explicit as possible.

5. Cost minimization: perfect substitutes. Suppose that a firm's production function is $y = f(x_1, x_2) = \alpha x_1 + \beta x_2$, defining all component terms as in the previous problem.

a) Fill in the missing spaces in the conditional factor demand function below.

$$x_1(w_1, w_2, y) = \begin{cases} \quad\quad\quad & \text{if} \quad > \\ & \text{otherwise} \end{cases} \qquad x_2(w_1, w_2, y) = \begin{cases} \quad\quad\quad & \text{if} \quad > \\ & \text{otherwise} \end{cases}$$

b) What is the cost of producing y units if you use only input 1? What about if you use only input 2? Use these answers to write general cost function, $C(y)$.