Third test, ECON 201, fall 2015. NAME: ___________________________________

Answer in the space provided. Show correct work for full credit. Box your final answers.

1. Perfect competition and monopoly. Suppose that the marginal cost of producing a particular good is $MC = 12$, and that there are no other costs. The market price is given by the inverse demand function $p(Y) = 48 - \frac{3}{10}Y$, where Y is the total quantity of output.

a) If this market is perfectly competitive, find the price (p^*) and quantity (Y^*) in equilibrium.

b) If there is only one seller, who is a profit maximizing monopolist, find the equilibrium price and quantity.

2. Duopoly. Assume the same marginal cost and inverse demand function as above, but now suppose that there are two firms who produce this good; firm 1 chooses an output quantity y_1, firm 2 chooses an output quantity y_2, and the resulting total output quantity is $Y = y_1 + y_2$.

a) Find the reaction functions $y_1^r(y_2)$ and $y_2^r(y_1)$, which give the profit-maximizing quantity of each firm, dependent on the other firm's quantity.

b) Find the Cournot equilibrium values of y_1^*, y_2^*, and p^*.

c) Find the Stackelberg equilibrium values of y_1^*, y_2^*, and p^*, assuming that firm 1 is the leader (decides its quantity first) and firm 2 is the follower (decides its quantity second).

3. Exchange. Arthur has 80 xylophones and zero yaks. Belinda has zero xylophones and 40 yaks. Arthur and Belinda have the utility functions $U_A = x_A y_A$, and $U_B = {x_B}^3 y_B$, respectively. (Here, x_A represents Arthur's consumption of xylophones, y_B represents Belinda's consumption of yaks, and so on.) Arthur and Belinda may trade xylophones for yaks, but the total number of each is fixed. Let $p \equiv p_x/p_y$ represent the ratio of the xylophone price to the yak price. Suppose that both Arthur and Belinda take the value of p as given.

a) Write down two equations in terms of x_A, y_A, and p which can be used to find Arthur's utility-maximizing quantities of xylophones and yaks when trading at a price ratio of p. Use these to solve for x_A in terms of p.

b) Repeat part (a) for Belinda, solving for x_B in terms of p.

c) Use your answers from parts (a) and (b) to solve for the value of p at which demand and supply are equal.

d) Use your answer from part (c) to find the numerical values of x_A, y_A, x_B, and y_B in the competitive equilibrium.

4. Quantity-based price discrimination. I write stories and sell them in Woodstock, NY, which is populated by a mix of awesome people and boring people. Each awesome person has the inverse demand curve $p^A(q^A) = 10 - q^A$, and each boring person has the inverse demand curve $p^B(q^B) = 6 - q^B$, where p and q represent the price and quantity of my stories. There are the same number of boring people as awesome people. I can supply stories with zero marginal cost. I can't tell the awesome people apart from the boring people. But I will charge customers for packages of multiple stories rather than charging by the individual story.

For parts a-c, suppose that I offer a 10-story package and a 6-story package, in exchange for fees F_{10} and F_6, respectively.

a) Find the profit-maximizing value of F_6, assuming that I want boring people to buy the small package.

b) Write the constraint on the price of the larger package, and use it to find the profit-maximizing value of F_{10}.

c) Find π, my profit from a representative group of one awesome person and one boring person.

In parts d-f, suppose that I offer a 10-story package and an x-story package, in exchange for fees F_{10} and F_x, respectively. Use the notation V_q^A and V_q^B to indicate the total value of a q-story package to a type A person and a type B person, respectively.

d) Find an expression for profit in terms of V_{10}^A, V_x^A, and V_x^B.

e) Use the expression from part (d) to solve for the profit-maximizing value of x.

f) Given the value of x you found in part e, calculate numerical values for F_{10}, F_x, and π.

5. Group-based price discrimination. I am still selling stories in Woodstock. My marginal cost is still zero, and my market still consists of awesome people who have inverse demand $p^A(q^A) = 10 - q^A$ and boring people who are equally numerous and who have inverse demand $p^B(q^B) = 6 - q^B$.

Unlike the previous problem, I can tell the difference between awesome people and boring people, and charge them different prices. Also unlike the previous problem, I will charge my customers by the story. Find the profit-maximizing prices and quantities for each type of consumer. Also, find π, my total profit from a representative group of two consumers.

$q^A =$ ____ $p^A =$ ____ $q^B =$ ____ $p^B =$ ____ $\pi =$ ____

Now the exam is over. Hooray! Nothing is happening in this space below. You can leave it blank, use it as scratch, or draw a picture of what you will do over the holiday break.