Econ 353: Public Choice

Partial answer key for problem set 2

Example 1	Example 2	Example 3	Example 4
20: A>B>C	20: A>B>C	25: A>B>C	49: A>B>C
25: A>C>B	35: A>C>B	10: B>A>C	48: B>A>C
35: B>C>A	25: B>C>A	25: B>C>A	3: C>B>A
20: C>B>A	20: C>B>A	40: C>A>B	

For all examples, assume sincere voting unless otherwise stated.

1-1. Positional rules. Find the plurality, Borda, and anti-plurality winners in example 1.

Plurality: A. Borda: B. Anti-plurality: C.

1-2. Elimination rules. Find the Hare, Baldwin, and Coombs winners in example 1.

Hare: eliminate C, elect B. Baldwin: eliminate A, elect B. Coombs: eliminate A, elect B.

1-3. Condorcet analysis. Construct a tournament diagram and pairwise matrix from example 1. Use this to find the Smith set, and the minimax, ranked pairs, beatpath, Condorcet-Hare, and Black winners.

B is a Condorcet winner. The Smith set is {B}. B wins all Condorcet consistent rules.

1-4. Strategy in plurality. Given example 1 and the plurality rule, is there a group of voters who can gain through strategy? Explain.

If the C>B>A voters vote for B, B wins instead of A. Or, if the B>C>A voters vote for C, C wins.

2-1. Positional rules. Find the plurality, Borda, and anti-plurality winners in example 2.

Plurality: A. Borda: A. Anti-plurality: C.

2-2. Elimination rules. Find the Hare, Baldwin, and Coombs winners in example 2.

Hare: eliminate C, elect A. Baldwin: eliminate B, elect A. Coombs: eliminate A, elect C.

2-3. Condorcet analysis. Construct a tournament diagram and pairwise matrix from example 2. Use this to find the Smith set, and the minimax, ranked pairs, beatpath, Condorcet-Hare, and Black winners.

A is a Condorcet winner. The Smith set is {A}. A wins all Condorcet consistent rules.

3-1. Positional rules. Find the plurality, Borda, and anti-plurality winners in example 3.

Plurality: C. Borda: C. Anti-plurality: A.

3-2. Elimination rules. Find the Hare, Baldwin, and Coombs winners in example 3.

Hare: eliminate A, elect B. Baldwin: eliminate B, elect C. Coombs: eliminate B, elect C.

3-3. Condorcet analysis. Construct a tournament diagram and pairwise matrix from example 3. Use this to find the Smith set, and the minimax, ranked pairs, beatpath, Condorcet-Hare, and Black winners.

The Smith is {A, B, C}. Minimax, ranked pairs and beatpath: C. Condorcet-Hare: B. Black: C.

3-4. Strategy. Explain why most voting rules will be vulnerable to strategy in example 3.

Whichever candidate wins, some majority prefers another candidate.

4-1. Positional rules. Find the plurality, Borda, and anti-plurality winners in example 4.

Plurality: A. Borda: B. Anti-plurality: B.

4-2. Elimination rules. Find the Hare, Baldwin, and Coombs winners in example 4.

Hare: eliminate C, elect B. Baldwin: eliminate C, elect B. Coombs: eliminate C, elect B.

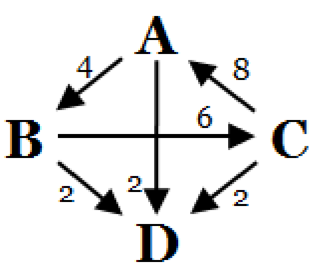
4-3. Condorcet analysis. Construct a tournament diagram and pairwise matrix from example 4. Use this to find the Smith set, and the minimax, ranked pairs, beatpath, Condorcet-Hare, and Black winners.

B is a Condorcet winner. The Smith set is {B}. B wins all Condorcet consistent rules.

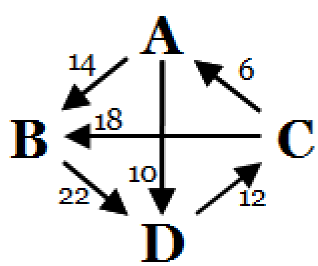
4-4. Strategy in Borda, minimax, Hare. Given example 4 and the Borda rule, is there a group of voters who can gain through strategy? How about minimax? How about Hare? Explain.

Borda or minimax: if the B>A>C voters vote B>C>A, B wins. Hare is non-manipulable.

<u>Example 5</u>



<u>Example 6</u>



In examples 5 and 6, each arrow in the tournament diagrams points from the winner to the loser of each pairwise comparison. The attendant number represents the margin of each pairwise victory.

5. Condorcet rule comparison. Find the Smith set and the minimax, beatpath, and ranked pairs winners in example 5.

Smith set: {A, B, C}. Minimax: D. Beatpath: B. Ranked pairs: B.

6. Condorcet rule comparison. Find the Smith set and the minimax, beatpath, and ranked pairs winners in example 6.

Smith set: {A, B, C, D}. Minimax: A. Beatpath: A. Ranked pairs: C.

Example 7

#	A	B	C
40	100	60	0
30	40	100	0
30	20	0	100

Example 8

#	A	B	C
45	100	0	0
30	0	100	80
25	0	80	100

For examples 7 and 8, let the entries in the # column represent a number of voters, let the entries in the A column represent the utilities of these voters for candidate A, and so on. Assume that a utility of 50 or more implies that a voter sincerely 'approves of' a candidate.

7. Approval vs range. Find the approval voting and range voting winners in example 7.

Approval: B. Range: A.

8. Strategy in approval and range. Find the approval voting and range voting winners given sincere voting in example 8, and explain why the outcome is hard to predict if voters behave strategically.

Approval: B or C. Range: B. The outcome depends on a chicken game between the B and C voters.

9. Majority criterion. Give an example in which Borda fails the majority criterion.

55: A>B>C. 45: B>C>A.

10. Majority and mutual majority criteria. Name two voting rules that pass the majority criterion but not the mutual majority criterion. Explain why each passes the majority criterion, and give an example in which each fails the mutual majority criterion.

Plurality, runoff, minimax.

11. Condorcet and Condorcet loser criteria. Name two voting rules that pass the Condorcet loser criterion but not the Condorcet criterion. Explain why each passes the Condorcet loser criterion, and give an example in which each fails the Condorcet criterion.

Borda, runoff, Hare.

12. Monotonicity criterion. Give an example in which Hare fails the monotonicity criterion. Explain why Borda and minimax both pass the monotonicity criterion.

13. Arrow theorem. What does the Arrow theorem demonstrate? Give as much intuition for the result as you can.

Non-dictatorship, unrestricted domain, independence of irrelevant alternatives, and Pareto can't be simultaneously satisfied in an algorithm that makes a social preference ordering from individual preference orderings.

14. Gibbard-Satterthwaite theorem. What does the Gibbard-Satterthwaite theorem demonstrate? Give as much intuition for the result as you can.

Non-dictatorship, unrestricted domain, and strategy-proofness can't be simultaneously satisfied.

<u>Example 15</u>: 120 voters. 2 seats to be filled. 81 voters for party A, and 39 voters for party B

15-1. Hare quota. Find the outcome of example 15 in a party list system with the Hare quota.

Quota is 60. One seat to each party.

A: 81 – 60 = 21

B: 39

15-2. Droop quota. Find the outcome of example 15 in a party list system with the Droop quota.

Quota is 40. Both seats to party A.

A: 81 – 40 = 41 – 40 = 1

B: 39

15-3. STV. If we have an STV system instead of a party list system, is the outcome different?

It's the same if voters are strictly party-loyal. Like 15-1 if Hare quota is used or 15-2 if Droop.

15-4. D'Hondt. Find the outcome of example 15 in a party list system using the D'Hondt method.

Both seats to party A.

A: 81 40.5 27

B: 39

15-5. Sainte-Laguë. Find the outcome of example 15 in a party list system using the Sainte-Laguë method.

One seat to each party.

A: 81 27

B: 39 13

15-6. Block voting. Discuss the outcome of example 15 if block voting is used.

If party A runs 2 candidates, and its voters split their votes evenly, each has 81 votes. Even if party B runs only one candidate, the most it can get is 78 votes. Both seats to party A in equilibrium.

15-7. SNTV or cumulative voting. Discuss the outcome of example 15 if SNTV or cumulative voting is used.

Similar to 15-6. Given even split of party A's votes, both seats to party A.

<u>Example 16</u>: 1100 voters. 10 seats to be filled. 501 voters for party A, 400 for B, and 199 for C.

16-1. Hare quota. Find the outcome of example 16 in a party list system with the Hare quota.

Quota: 110. To parties A, B, and C: 4, 4, and 2 seats, respectively.

A: 501 – 110 = 391 – 110 = 281 – 110 = 171 – 110 = 61

B: 400 – 110 = 290 – 110 = 180 – 110 = 70

C: 199 – 110 = 89

16-2. Droop quota. Find the outcome of example 16 in a party list system with the Droop quota.

Quota: 100. To parties A, B, and C: 5, 4, and 1 seats, respectively.

A: 501 – 100 = 401 – 100 = 301 – 100 = 201 – 100 = 101 – 100 = 1

B: 400 – 100 = 300 – 100 = 200 – 100 = 100 – 100 = 0

C: 199 – 100 = 99

16-3. STV. If we have an STV system instead of a party list system, is the outcome different?

It's the same if voters are strictly party-loyal. Like 16-1 if Hare quota is used or 16-2 if Droop.

16-4. D'Hondt. Find the outcome of example 16 in a party list system using the D'Hondt method.

For parties A, B, and C: 5, 4, and 1 seat, respectively.

A: 501	*250.5*	*167*	*125.5*	*100.2*
B: 400	*200*	*133.3*	*100*	
C: 199				

16-5. Sainte-Laguë. Find the outcome of example 16 in a party list system using the Sainte-Laguë method.

For parties A, B, and C: 4, 4, and 2 seats, respectively.

A: 501	*167*	*100.2*	*71.6*
B: 400	*133.3*	*80*	*57.1*
C: 199	*66.3*		

16-6. Block voting. Discuss the outcome of example 16 if block voting is used.

Equilibrium.: A: 5 candidates, 100.2 votes each. B: 4 candidates, 100 votes each. C: 1 or 2 candidates. For parties A, B, and C: 5, 4, and 1 seats, respectively.

16-7. SNTV or cumulative voting. Discuss the outcome of example 15 if SNTV or cumulative voting is used.

Similar to 16-6. For parties, A, B, and C: 5, 4, and 1 seats, respectively.

Example 17: 100 voters. 3 seats to be filled.

31: $A_1 > A_2 > A_3 > B_1 > B_2 > B_3$

20: $A_2 > A_1 > A_3 > B_1 > B_2 > B_3$

10: $A_3 > A_1 > A_2 > B_1 > B_2 > B_3$

20: $B_1 > B_2 > B_3 > A_1 > A_2 > A_3$

15: $B_2 > B_1 > B_3 > A_1 > A_2 > A_3$

4: $B_3 > B_1 > B_2 > A_1 > A_2 > A_3$

17. STV. Find the outcome of example 17 in an STV system with the Droop quota.

Quota: 25. Candidates A_1, A_2, and B_1 are elected.

A_1	A_2	A_3	B_1	B_2	B_3
31	*20*	*10*	*20*	*15*	*4*
✓	*+6*				
[25]	*26*	*10*	*20*	*15*	*4*
	✓	*+1*			
[25]	*[25]*	*11*	*20*	*15*	*4*
			+4		✗
[25]	*[25]*	*11*	*24*	*15*	*[0]*
		✗	*+11*		
[25]	*[25]*	*[0]*	*35*	*15*	*[0]*
			✓		

18. Party list. Explain the difference between an open party list system and a closed party list system

19. MMP and parallel. Explain the difference between a regular party list system, a mixed member proportional system, and a parallel system.

20. Proxy systems. Explain how proxy systems differ from traditional proportional representation systems.