Problem set 3: Taxation and Optimal Government Size. Due April 2nd, 2015

1. Optimal government size, example 1. Suppose that there is a private good, with a market that is efficient in the absence of taxation, and a public good that can only be provided by the government using revenue from a per-unit tax of τ on the private good. Let the marginal benefit and cost for the private good (with quantity x) be as follows:

$$MB(x) = 90 - x \qquad MC(x) = 10 + x$$

Let the marginal benefit and cost of the public good (with quantity y) be as follows:

$$MB(y) = 120 - 4y \qquad MC(y) = 40$$

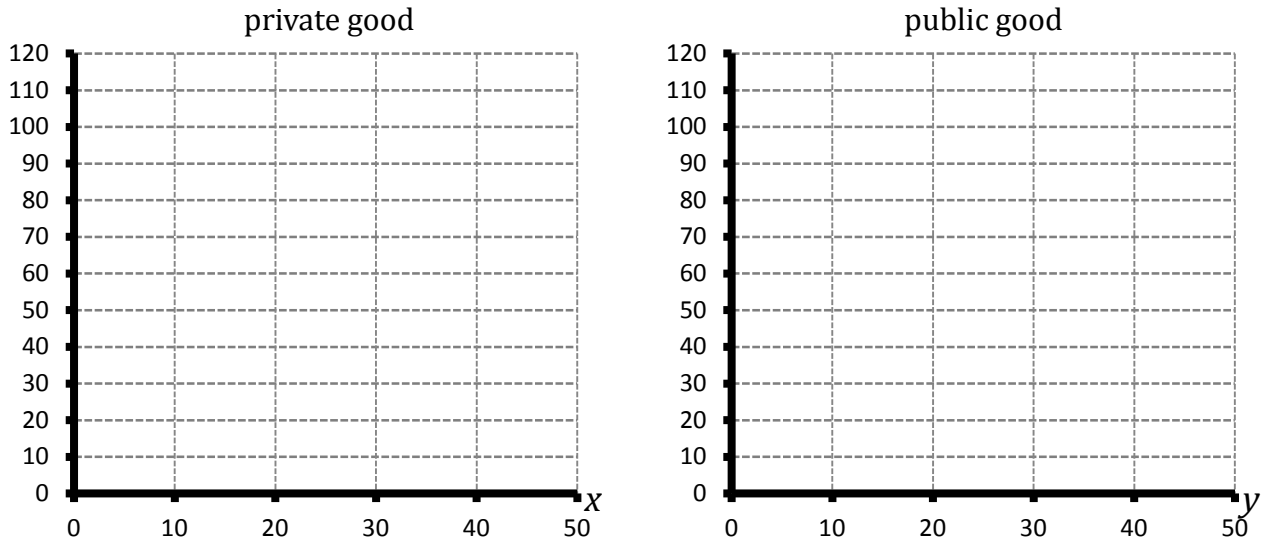
a) As functions of the tax τ, find the quantity of the private good $x(\tau)$, tax revenue $R(\tau)$, and deadweight loss $D(\tau)$.

b) Find the optimal tax τ^o, supposing that the shadow value of spending a dollar on the public good is $\lambda = 3/2$. Also, find the resulting tax revenue, $R(\tau^o)$.

c) Using the value of tax revenue that you found, and information about the public goods market above, demonstrate that the shadow value of the last dollar spend on the public good is indeed $3/2$. That is, find the value of y^o that results from spending the revenue $R(\tau^o)$, and use this to calculate the shadow value $\lambda(y^o)$.

d) Compare the deadweight loss in the private market resulting from τ^o to the gain in surplus in the public market resulting from y^o. How much economic surplus is gained overall from the tax and the spending *taken together*?

e) On the left, graph the market for the private good, with the optimal tax, labeling CS, PS, R, and D. On the right, graph the market for the public good, labeling R (government expenditure, equal to revenue by assumption) and TES_y (total economic surplus from the public good).



2. Optimal government size, derivations. Like the previous question, but with more general parameters:

$$MB(x) = \alpha - \beta x \qquad MC(x) = \gamma + \delta x$$
$$MB(y) = \mathrm{A} - \mathrm{B}y \qquad MC(y) = \Gamma$$

a) On the side of the private good market, derive $x(\tau)$, $R(\tau)$, $D(\tau)$, and τ^o, in terms of α, β, γ, δ, and λ.

b) On the side of the public good market, derive the optimal amount of the public good, y^o, as a function of A, B, Γ, and μ, where μ is the shadow cost of the last dollar of tax revenue.

3. Optimal government size, example 2. Suppose that there is a private good, with a market that is efficient in the absence of taxation, and a public good that can only be provided by the government using revenue from a per-unit tax of τ on the private good. Let the marginal benefit and cost for the private good (with quantity x) be as follows:

$$MB(x) = 150 - 3x \qquad MC(x) = 30 + x$$

Let the marginal benefit and cost of the public good (with quantity y) be as follows:

$$MB(y) = 160 - 8y \qquad MC(y) = 48$$

a) As functions of the tax τ, find the quantity of the private good $x(\tau)$, tax revenue $R(\tau)$, and deadweight loss $DWL(\tau)$.

b) Find the optimal tax τ^o, supposing that the shadow value of spending a dollar on the public good is $\lambda = 4/3$. Also, find the resulting tax revenue, $R(\tau^o)$.

c) Using the value of tax revenue that you found, and information about the public goods market above, demonstrate that the shadow value of the last dollar spend on the public good is indeed $4/3$. That is, find the value of y^o that results from spending the revenue $R(\tau^o)$, and use this to calculate the shadow value $\lambda(y^o)$.

d) In the above model, λ represents the shadow value of the last dollar of public spending, and μ represents the shadow cost of the last dollar of tax revenue. In this model, what is the relationship between these variables when government is 'too small', i.e. below its optimal size? Explain your answer clearly.

e) Discuss the Laffer curve in the context of the above example. What levels of taxation are Pareto-dominated regardless of the value of public spending?

4. Simple income tax calculation. Suppose that there is a graduated income tax defined by this table:

income bracket	marginal rate
\$0 - \$10,000	10%
\$10,000 - \$20,000	20%
\$20,000 - \$30,000	30%
\$30,000 and above	40%

If your taxable income is \$25,000, how much do you pay? _____

If your taxable income is \$50,000, how much do you pay? _____

5. Progressivty and regressivity of simple tax structures. Let Y be a person's taxable income, and let $T(Y)$ be the person's total tax liability as a function of income. For each of the tax functions below, carefully and rigorously discuss its vertical equity properties; that is, is it progressive, regressive, or proportional, and does it matter what income range we use in our comparison? Also, for functions (b) and (c), identify a tax levied by the US government that has a similar structure.

a) $T(Y) = \kappa + mY$, where $\kappa < 0$ and $m \in (0, 1)$.

b) $T(Y) = \min\{mC, mY\}$, where $C > 0$ and $m \in (0, 1)$.

c) $T(Y) = \max\{0, m(Y - E)\}$, where $E > 0$ and $m \in (0, 1)$.