Problem Set 1: Public Goods. Due Monday, February 22nd, 2016

1. Five roommates are deciding how much gunpowder to buy for the defense of their dorm suite. The marginal cost of gunpowder, in pounds, is $MC = 10$.

1.1. First, let's consider what happens if the roommates have identical preferences about the public good, which can be represented by the marginal benefit function $MB_i = 20 - \frac{1}{5}y$, where y is the quantity of gunpowder they buy, in pounds.

a) In the absence of coordination, the Nash equilibrium quantity of gunpowder is ________.

b) The Pareto optimal quantity of gunpowder is ________.

c) Explain the outcome of this case given a majority voting equilibrium with equal cost division, and a Lindahl tax scheme.

1.2. Now, let's consider an alternative case with heterogeneous preferences. Specifically, suppose that the first four roommates each have the marginal benefit functions $MB_i = 8 - \frac{1}{3}y$, but the fifth has the marginal benefit function $MB_5 = 28 - \frac{7}{6}y$.

c) In the absence of coordination, the Nash equilibrium quantity of gunpowder is ________ (This may not be a whole number, so you can approximate it or express it as a fraction.)

d) The Pareto optimal quantity of gunpowder is ________.

e) If the roommates agree to divide the cost evenly, and then decide the quantity via a majority voting process, the equilibrium quantity will be ________.

f) In an ideal Lindahl scheme, how would the roommates divide the cost of gunpowder? How much gunpowder would they buy?

2. The People's Republic of Blargsburg contains five people, named Person 1, Person 2, etc. They are deciding how big a statue to build to honor the memory of their glorious founder, Chairman Blarg. All residents of Blargsburg love Chairman Blarg, but some love him more than others. Thus, the marginal benefit functions for the five people are different:

$$MB_1 = \frac{30}{y} \qquad MB_2 = \frac{90}{y} \qquad MB_3 = \frac{140}{y} \qquad MB_4 = \frac{210}{y} \qquad MB_5 = \frac{300}{y}$$

where y is the height of the statue, in feet. The marginal cost of each next foot of statue height is $MC = 10$.

a) If the people are purely selfish and must contribute to the statue fund in an uncoordinated way, the Nash equilibrium height of the statue is ________ feet.

b) The Pareto optimal statue height is ________ feet.

c) Suppose that it is agreed that each citizen will pay $1/5$ of the cost of the statue, and the height of the statue will be determined by majority rule. Given this agreement, person 1's first choice of statue height is $y_1^* =$________ feet. Person 2's first choice is $y_2^* =$________ feet. Similarly, we have $y_3^* =$________, $y_4^* =$________, and $y_5^* =$________. The unique equilibrium in majority voting is $y_{mv}^* =$________.

d) Explain why y_{mv}^* is a majority voting equilibrium.

e) Compare the Nash equilibrium without coordination in part (a) and the majority voting equilibrium in part (c), in terms of how close they come to the Pareto optimum. Provide some intuition for what you find.

f) If you know the marginal benefit functions of all individuals, you could assign the Lindahl shares $s_1 =$________, $s_2 =$________, $s_3 =$________, $s_4 =$________, and $s_5 =$________.

g) Show that, if you assign people these Lindahl shares instead of the equal cost division as in part (c), the majority voting equilibrium will result in a Pareto optimal statue height.