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Public Economics, spring 2016

Adverse selection exercise

Definitions

Suppose that potential buyers of insurance differ from one another in terms of risk aversion and in terms of health. Let individuals be divided into $I \times J$ distinct types, where I is the number of risk aversion types, and J is the number of health types.

Let $c = [c_1 \quad \cdots \quad c_J]$ be the cost vector. That is, let c_j represent the expected value of the medical costs of a person with health type j. Assume that these costs are exogenous, i.e. independent of any choice that the person makes.

Let $b = \begin{bmatrix} b_{11} & \cdots & b_{1J} \\ \vdots & \ddots & \vdots \\ b_{I1} & \cdots & b_{IJ} \end{bmatrix}$ be the benefit matrix. That is, let b_{ij} represent the ex-ante willingness to pay for insurance for a person with risk aversion type i and health type j. This willingness to pay depends positively on both the person's expected medical costs and their level of risk aversion.

Let $x = \begin{bmatrix} x_{11} & \cdots & x_{1J} \\ \vdots & \ddots & \vdots \\ x_{I1} & \cdots & x_{IJ} \end{bmatrix}$ be the participation matrix. That is, let x_{ij} be equal to one if a person with risk aversion type i and health type j buys health insurance, and let it be equal to zero otherwise.

Suppose for now that each type is equally common in the population.

Equilibrium under asymmetric information

Suppose that health insurance companies cannot observe the health type or the risk aversion type of each potential customer, though individuals know their own types.

Suppose that the health insurance market is perfectly competitive.

An equilibrium is defined by a participation matrix x^* and a premium price p^* such that $x_{ij}^* = \begin{cases} 1 & \text{if } b_{ij} \geq p^* \\ 0 & \text{otherwise} \end{cases}$, and p^* is greater than or equal to the expected value of claims for a person selected at random from those types that *are* participating in the market.

Multiple equilibria may exist, but we will focus on only the equilibria that include the greatest possible number of types.

We can evaluate each equilibrium in terms of D, the deadweight loss resulting from the information asymmetry, by counting the difference between the potential benefits net of costs of insuring those types of people who are *not* insured in the equilibrium.

Example 1

Setup: $c = [12 \quad 24 \quad 36]$ $\quad b = \begin{bmatrix} 15 & 29 & 39 \\ 18 & 35 & 42 \end{bmatrix}$

Solution…

Consider first the possibility of an equilibrium in which all six types purchase insurance, i.e. a participation matrix of $x = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. In this case, the expected cost to the insurance company of issuing a policy is $\frac{12+24+36}{3} = 24$, which means that the equilibrium price of a policy must be greater than or equal to 24, because otherwise insurance companies are making negative profit on each policy. Unfortunately, the healthiest people will be unwilling to buy insurance at such a price, because their benefits are less than this.

Thus, we update the participation matrix to $x' = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Might this updated participation matrix be an equilibrium? In this case, the expected cost of issuing a policy is $\frac{24+36}{2} = 30$. At a price of 30 or greater, a person with $i = 1$ and $j = 2$ will not be willing to participate.

So, we update the participation matrix again, to $x'' = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, and check to see if this might be an equilibrium. In this case, the expected cost of a policy is $\frac{24+36+36}{3} = 32$. Happily, all of the people with types indicated by x'' are willing to pay this cost. (That is, $x''_{ij} = 1 \rightarrow b_{ij} > 32$.) So, we have equilibria in which $x^* = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, and $p^* \in [32, 35]$.

In these equilibria, none of the healthiest people buy insurance, all of the sickest people buy insurance, and of the middle-health people, only the more risk-averse buy insurance. We calculate the deadweight loss as $D = (15 - 12) + (18 - 12) + (29 - 24) = 14$.

Example 2

Setup: $c = [30 \quad 50]$ $\quad b = \begin{bmatrix} 35 & 55 \\ 41 & 61 \\ 47 & 67 \end{bmatrix}$

Solution: $x^* = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ $\quad p^* \in [45, 47]$ $\quad D = 16$

In these equilibria, only the most risk-averse of the healthy people buy insurance.

Example 3

Setup: $c = [12 \quad 24]$ $\quad b = \begin{bmatrix} 15 & 30 \\ 19 & 40 \end{bmatrix}$

Solution: $x^* = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ $\quad p^* \in [24, 30]$ $\quad D = 10$

In these equilibria, only the sick people buy insurance.