James Green-Armytage

Public Economics, spring 2016

Rawlsian redistribution exercise

Introduction

In *A Theory of Justice*, John Rawls argues that a 'just' amount of redistribution would be that which people would decide on if they didn't know whether they would be rich or poor. In microeconomic theory, we have a theoretical framework in which individuals maximize their 'expected utility' of consumption, given some uncertainty about which 'state of the world' will occur. Here, we will use microeconomic tools to model a highly simplified version of Rawls's problem, in which there are only two possible values of pre-redistribution income, and behavioral responses to taxation are represented in a stylized way by a single 'leaky bucket' parameter.

Model

Suppose that you are with others behind a Rawlsian 'veil of ignorance', deciding how much redistribution there should be in your society. Once the decision is made, you will find out whether you are a poor person, with pre-redistribution wealth w_P, or a rich person, with pre-redistribution wealth w_R. The probability of these are π_P and π_R, respectively; thus, the ratio of rich people to poor people in the society will be π_R/π_P.

Unfortunately, redistribution is 'leaky'. That is, for every \$1 of wealth that is taxed away from a rich person, only \$$\delta$ reaches the poor, where $\delta \in (0, 1)$; the rest is simply wasted. We use this to represent the principle that redistribution is typically accompanied by deadweight loss, such that the economic surplus gained by the poor (where 'economic surplus' is measured in terms of dollars, not in terms of utility) is somewhat less than the economic surplus lost by the rich. Let x be the amount of money taxed away from each rich person.

Let c_R be the consumption of a rich person, i.e. is his or her starting wealth, minus the amount that he or she must pay in taxes. That is,

$$c_R = w_R - x$$

To find c_P, the consumption of a poor person, we need to use both the 'leakiness parameter' δ, and the ratio of poor people to rich people, which we found above to be π_R/π_P. That is, if each rich person pays x, the total post-leak tax revenue is δx, multiplied by the number of rich people. This must be divided equally among the poor people, so each poor person must receive δx, multiplied by the ratio of rich to poor people. So, each poor person's consumption is

$$c_P = w_P + \frac{\pi_R}{\pi_P}\delta x$$

Your Von Neumann-Morgenstern utility function is $V(c)$, where c represents your consumption in either state of the world. Therefore, your expected post-redistribution utility is

$$U = \pi_P V(c_P) + \pi_R V(c_R)$$

where c_P and c_R represent your consumption if poor and if rich, respectively. We assume that $V'(c) > 0$ and $V''(c) < 0$.

Your objective is to choose x to maximize this expected utility, taking w_P, w_R, π_P, π_R, and δ as given; that is, you are choosing how much redistribution you would like there to be, when you don't know whether you will be the benefactor or the beneficiary of this redistribution.

Solving by analogy to elementary consumer theory

In previous units, we practiced solving optimization problems in which a consumer maximized $U(x_1, x_2)$ subject to the constraint that $p_1x_1 + p_2x_2 \leq m$; there, x_1 and x_2 were quantities of goods, p_1 and p_2 were prices, and m was an 'endowment' of income. Given strictly convex preferences, we found that optimal consumption solved the equations $MU_1/MU_2 = p_1/p_2$ and $p_1x_1 + p_2x_2 = m$. We can harness this result by making our current problem look more like this one. We already have a utility function $U(c_P, c_R) = \pi_P V(c_P) + \pi_R V(c_R)$ that can be readily adapted to this purpose, but instead of a single budget constraint, we have two separate equations that define c_P and c_R.

As it turns out, we can transform these into a single budget constraint with just a little algebra. We begin with

$$c_P = w_P + \frac{\pi_R}{\pi_P}\delta x \qquad\qquad c_R = w_R - x$$

Let's multiply both sides of the second equation by $\frac{\pi_R}{\pi_P}\delta x$, so that the xs will cancel when we add the two equations together:

$$c_P = w_P + \frac{\pi_R}{\pi_P}\delta x \qquad\qquad \frac{\pi_R}{\pi_P}\delta c_R = \frac{\pi_R}{\pi_P}\delta w_R - \frac{\pi_R}{\pi_P}\delta x$$

Adding the two equations, we obtain:

$$\boxed{c_P + \frac{\pi_R}{\pi_P}\delta c_R = w_P + \frac{\pi_R}{\pi_P}\delta w_R}$$

This is what we were aiming for. With the variables rearranged in this form, we have a single budget constraint that reveals the 'price ratio' of consumption in the two possible states of the world. That is, looking at the left-hand side, we can see that in order to get an extra unit of consumption in the 'poor' state, we must give up $\frac{\pi_R}{\pi_P}\delta$ units of consumption in the 'rich' state.

So, given that our preferences over c_P and c_R are strictly convex (which isn't hard to verify, given our assumptions that $V'(c) > 0$ and $V''(c) < 0$), we will have optimal redistribution when this 'price ratio' is equal to our equivalent of the marginal rate of substitution, i.e. $(\partial U/\partial c_P)/(\partial U/\partial c_R)$, which we can find as follows:

$$U = \pi_P V(c_P) + \pi_R V(c_R)$$

$$\frac{\left(\frac{\partial U}{\partial c_P}\right)}{\left(\frac{\partial U}{\partial c_R}\right)} = \frac{\pi_P V'(c_P)}{\pi_R V'(c_R)}$$

Setting this equal to the 'price ratio' from above, we have

$$\frac{1}{\left(\frac{\pi_R}{\pi_P}\delta\right)} = \frac{\pi_P V'(c_P)}{\pi_R V'(c_R)}$$

$$\frac{\pi_P}{\pi_R} = \delta\frac{\pi_P V'(c_P)}{\pi_R V'(c_R)}$$

$$\boxed{\frac{V'(c_R)}{V'(c_P)} = \delta} \qquad \text{or} \qquad \boxed{V'(c_R) = \delta V'(c_P)}$$

When combined with the budget constraint (above, also boxed), this equation can be used to find the optimal value of x, and the resulting values of c_P and c_R. Further, it is worthy of careful consideration in its own right. It says that the ratio of marginal utilities of consumption in the two states should be equal to the value of the 'leakiness' parameter, i.e. the share of redistributed wealth that survives the redistribution process.

For example, if δ is precisely 1, we have $V'(c_R) = V'(c_P)$ and thus $c_R = c_P$. That is, if there is no inefficiency (or 'leakiness') whatsoever in the redistribution process, we may as well continue redistributing until we will have precisely the same post-redistribution consumption whether we start as 'rich' or 'poor'.

However, we have assumed that $\delta < 1$, which is to say that redistribution results in some inefficiency. Therefore, when the $V'(c_R) = \delta V'(c_P)$ equation holds, the marginal utility of consumption for the rich must be less than the marginal utility of consumption for the poor.

Since we are assuming a strictly concave von Neumann-Morganstern utility function $V(c)$, this implies that the rich must remain richer than the poor, even after optimal redistribution takes place. Furthermore, the extent of post-redistribution inequality will be greater when redistribution is less efficient, i.e. when δ is smaller. These results are intuitive.

Alternative solution

Our problem here is to choose x (redistribution) so as to maximize expected utility of consumption in two possible states of the world, i.e. $U(c_P, c_R)$. In the preceding discussion, we used algebra to fit this problem into the familiar framework of maximizing this utility function subject to a single budget constraint, which we found by combining our expressions for c_P and c_R, eliminating the x variable in the process. An alternative solution method is to re-write our maximization problem entirely in terms of x, and then solve using the condition $\partial U/\partial x = 0$.

Our basic statement of the maximization problem is

$$\max_x U = \pi_P V(c_P) + \pi_R V(c_R)$$

Plugging in our expressions for c_P and c_R, we can rewrite this as:

$$\max_x U = \pi_P V\left(w_P + \frac{\pi_R}{\pi_P}\delta x\right) + \pi_R V(w_R - x)$$

Setting the derivative of U with respect to x equal to zero, we find:

$$\frac{\partial U}{\partial x} = \pi_P V'(c_P)\left(\frac{\pi_R}{\pi_P}\delta\right) - \pi_R V'(c_R) \overset{\text{set}}{=} 0$$

$$\pi_R \delta V'(c_P) - \pi_R V'(c_R) = 0$$

$$V'(c_R) = \delta V'(c_P)$$

This is just as before. We can make this discussion more complete by verifying that the second derivative of U with respect to x is negative, so that U is maximized when $\partial U/\partial x = 0$:

$$\frac{\partial^2 U}{\partial^2 x} = \frac{\delta^2 {\pi_R}^2}{\pi_P} V''(c_P) + \pi_R V''(c_R)$$

Since we have assumed that $V''(c) < 0$, the above expression is indeed negative.

Example

Suppose that $\pi_P = 3/4$, $\pi_R = 1/4$, $\delta = 1/2$, $w_P = 40$, $w_R = 360$, and $V(c) = \ln c$. Using our first approach, we can start by re-writing our general one-equation budget constraint, and then plugging in the parameter values:

$$c_P + \frac{\pi_R}{\pi_P}\delta c_R = w_P + \frac{\pi_R}{\pi_P}\delta w_R$$

$$c_P + \frac{1}{6}c_R = 100$$

Similarly, we can adapt our general first order condition to the specifics of the example:

$$V'(c_R) = \delta V'(c_P)$$

$$\frac{1}{c_R} = \left(\frac{1}{2}\right)\left(\frac{1}{c_P}\right)$$

$$c_R = 2c_P$$

Plugging this expression for c_R back into the budget constraint, we have

$$c_P + \frac{1}{6}(2c_P) = 100$$

$$\frac{4}{3}c_P = 100$$

$$\boxed{c_P = 75}$$

$$\boxed{c_R = 150}$$

From here, we can work backwards to find x, the tax on each rich person. We know that $c_R = w_R - x$, and thus that $150 = 360 - x$, so we must have

$$\boxed{x = 210}$$

We may also use the alternative approach to solve for c_P, c_R, and x as follows. As we found above, this approach also leads us to the first order condition $V'(c_R) = \delta V'(c_P)$, which in this example gives us the equation $c_R = 2c_P$. In addition, we have the two other equations that define c_P and c_R in terms of x, giving us three equations and three unknowns:

$$c_R = w_R - x \qquad c_P = w_P + \frac{\pi_R}{\pi_P}\delta x \qquad V'(c_R) = \delta V'(c_P)$$

$$c_R = 360 - x \qquad c_P = 40 + \frac{1}{6}x \qquad c_R = 2c_P$$

Plugging the third equation into the first, and multiplying the second equation by 6, we have

$$2c_P = 360 - x \qquad 6c_P = 240 + x$$

Adding these two equations, we find that $8c_P = 600$. Thus, we confirm that $c_P = 75$, $c_R = 150$, and $x = 210$.