Supplemental appendix for

"Selecting the runoff pair"

This Supplemental Appendix presents additional results, in three parts. Section B.1 defines four additional criteria, and evaluates 23 runoff pair selection rules by these criteria using the Politbarometer data. Section B.2 varies the parameters of the spatial model, as a robustness check for the spatial model results in the main body of the paper. Section B.3 compares the strategy results of six runoff pair selection rules to six analogous one-round voting rules.

B.1. Additional criteria

B.1.1. Condorcet efficiency (CE) refers to the share of trials in which a Condorcet winner is included in the runoff pair. When there is no Condorcet winner for the trial, all rules receive a score of zero for that trial. This criterion favors rules that are more likely to choose at least one centrist candidate as a member of the runoff pair.

B.1.2. Double Condorcet efficiency (2CE) refers to the share of trials in which the runoff pair consists of a Condorcet winner, and a candidate who becomes the Condorcet winner when the first candidate is removed. When no such pair exists, all rules receive a zero score for that trial. This criterion favors rules that are more likely to set up a final election between two centrist candidates.

B.1.3. Average utility from designated representative (AUDR) divides voters between the two members of the chosen runoff pair such that the average of voter utilities for their "designated representative" is maximized. This average is the value of the statistic. This criterion is an alternative measure of "representativeness;" unlike the Rep criterion above, it is somewhat

sensitive to the later preferences of a majority whose favorite candidate has already been selected as the first member of the runoff pair.

B.1.4. Vote share of the loser (VSL) measures the fraction of votes that the losing candidate is expected to receive in the runoff election. Higher values of VSL indicate that rules will more often lead to closely competitive elections.

B.1.5. Results by additional criteria

Table B1 gives the Politbarometer results for CE, 2CE, AUDR, and VSL.

Eight of the 23 rules are Condorcet-efficient, and thus receive the maximum CE score, which is equal to the share of trials where a Condorcet winner exists. In this particular data set, that share is approximately 99.3%.[1]

Two of the 23 rules axiomatically achieve the maximum 2CE score: repeated Condorcet-Hare and repeated Black. In this data set, the maximum 2CE score is approximately 98.0%.

B.2. Effects of varying the spatial model parameters

Tables A5 and A6 provide information about the effects of varying the number of candidates and the number of voters in the spatial model, respectively.

In these simulations, the UEW, UEL, and representativeness scores of all nine rules are increasing and concave in the number of candidates. The plurality rule gains less by all three criteria than the other eight rules; thus its relative performance is worse when the number of candidates is large. Plurality exchanges the last ranking with CHUC by the UEL criterion, but otherwise there are no major reversals.

[1] A comparably high Condorcet winner rate of 98.7% occurs in the spatial model as well. In general, a spatial model with utility determined by distance and voters selected from a distribution with point symmetry will have Condorcet winners in 100% of the elections in the limit as the number of voters approaches infinity, because the candidate who is closest to the mode will beat all others in paired comparisons.

Intuitively, as the number of candidates increases, the greater competition helps to drive outcomes that are superior in terms of utility. But this effect is weakest for the plurality runoff rule, which encounters greater problems with vote splitting as the number of candidates grows.

In these simulations, UEW scores decline as the number of voters increases, with one slight exception. Most UEL scores decline as the number of voters increases, and all MU scores decline as the number of voters increases. In all three cases, the effect of additional voters grows weaker as the number of voters increases. Again, there are few dramatic reversals in the relative scores of the rules. As the number of voters increases, MRCH's UEL score increases so that it overtakes STV, and its representativeness score declines slowly, so that it exchanges places with plurality, but there are few other changes in the rankings.

Intuitively, when the number of voters is small, it is easier for a greater share of them to be satisfied by available candidates. As the number of voters grows, the model approaches a limiting case where the voters form a continuous distribution.

B.3. Runoff pair selection rules and voter strategy

Finally, Table A7 compares the strategic resistance (SR) scores of six runoff pair selection rules with six analogous rules that use only one round of voting. The six runoff pair selection rules are plurality, Hare, Condorcet-Hare, Borda, range, and approval, all as defined in Section 2 of the main paper. (We use the repeated version of Condorcet-Hare, because it has the highest strategic resistance scores.) The six other rules are the more familiar one-round implementations of the base rules used by each of these.

According to the Politbarometer data, four of the six runoff rules have greater strategic resistance than their one-round counterparts; the exceptions are Hare and Borda. According to the spatial model data, five of the six runoff rules out-perform their one-round counterparts; in this case, the only exception is Hare.

Intuitively, when the sincere winner of a runoff pair selection rule is also the sincere Condorcet winner, strategists will only be able to succeed by excluding this candidate from the runoff pair altogether, which is often a more difficult task than simply causing another candidate to win according to the one-round base rule. Thus if societies are particularly concerned about strategic voting, this difficulty in implementing a successful strategy might give them an additional reason to prefer a runoff system to a one-round system.

Table B1. Politbarometer results by additional criteria

Rank	CE		2CE		AUDR		VSL	
	Rule	Score	Rule	Score	Rule	Score	Rule	Score
1	R Condorcet-Hare	0.9934	R Condorcet-Hare	0.9803	MRCH	0.8118	CHCC	0.4076
2	CHUC	0.9934	R Black	0.9803	Hare	0.8118	CPO-STV	0.4041
3	MRCH	0.9934	Borda	0.9033	CHBC	0.8114	CHC-CPO-STV	0.4040
4	CHC-CPO-STV	0.9934	R Hare	0.9033	CHUC	0.8113	STV	0.4038
5	CHC-STV	0.9934	R Borda	0.8934	BBC	0.8112	CHC-STV	0.4038
6	CHBC	0.9934	norm range	0.8869	RUC	0.8109	Borda	0.4023
7	CHCC	0.9934	range	0.8574	STV	0.8100	Hare	0.4019
8	R Black	0.9934	approval	0.8230	CPO-STV	0.8098	R Condorcet-Hare	0.4018
9	Borda	0.9902	CHCC	0.7770	CHC-CPO-STV	0.8093	R Black	0.4018
10	R Borda	0.9885	CHC-CPO-STV	0.7443	CHC-STV	0.8091	R Hare	0.4013
11	norm range	0.9869	CHC-STV	0.7311	plurality	0.8082	norm range	0.4012
12	R Hare	0.9836	CPO-STV	0.7279	CHCC	0.8064	R Borda	0.4009
13	range	0.9787	R plurality	0.7115	MR range	0.8052	range	0.4001
14	BBC	0.9770	STV	0.6951	MR approval	0.8045	plurality	0.3991
15	approval	0.9754	Hare	0.6082	Borda	0.8008	MRCH	0.3975
16	RUC	0.9656	plurality	0.5918	norm range	0.8006	approval	0.3973
17	CPO-STV	0.9590	MRCH	0.5607	Range	0.8006	R plurality	0.3942
18	STV	0.9557	CHBC	0.5197	R Hare	0.8002	CHBC	0.3917
19	plurality	0.9459	BBC	0.5197	R Condorcet-Hare	0.8002	BBC	0.3916
20	Hare	0.9459	MR range	0.4508	R Black	0.8001	RUC	0.3881
21	MR range	0.9443	RUC	0.4443	R Borda	0.8000	CHUC	0.3869
22	R plurality	0.9361	CHUC	0.4426	approval	0.7984	MR approval	0.3731
23	MR approval	0.9213	MR approval	0.3836	R plurality	0.7973	MR range	0.3671

Abbreviations

CE = Condorcet efficiency
2CE = double Condorcet efficiency
AUDR = avg. util. from designated rep.
VSL = vote share of the loser
R = repeated
MR = modified repeated
CH = Condorcet-Hare
CHC = CH-constrained
CPO = comparison of pairs of outcomes
STV = single transferable vote
UC = utility complement
BC = Borda complement
CC = closest competitor
norm = normalized

Table B2. Spatial model results with variable number of candidates

	Number of candidates				
	3	5	7	9	11
UEW Scores					
Plurality	0.5917	0.6155	0.6251	0.6293	0.6302
Hare	0.5917	0.6166	0.6282	0.6345	0.6375
STV	0.5919	0.6169	0.6287	0.6351	0.6381
Condorcet-Hare	0.5921	0.6181	0.6311	0.6393	0.6448
CHUC	0.5921	0.6181	0.6311	0.6392	0.6448
MRCH	0.5921	0.6181	0.6311	0.6393	0.6448
Borda	0.5922	0.6181	0.6312	0.6393	0.6449
Range	0.5922	0.6182	0.6313	0.6394	0.6450
Approval	0.5921	0.6181	0.6309	0.6389	0.6445
UEL Scores					
Plurality	0.4983	0.5449	0.5598	0.5660	0.5672
Hare	0.4983	0.5502	0.5722	0.5846	0.5909
STV	0.5065	0.5578	0.5790	0.5901	0.5958
Condorcet-Hare	0.5091	0.5667	0.5926	0.6077	0.6177
CHUC	0.4865	0.5314	0.5527	0.5650	0.5715
MRCH	0.4994	0.5538	0.5791	0.5942	0.6037
Borda	0.5086	0.5660	0.5919	0.6072	0.6172
Range	0.5096	0.5677	0.5937	0.6089	0.6190
Approval	0.5085	0.5662	0.5912	0.6057	0.6151
Rep Scores					
Plurality	0.6647	0.6931	0.7026	0.7059	0.7060
Hare	0.6647	0.6955	0.7083	0.7150	0.7187
STV	0.6623	0.6931	0.7059	0.7130	0.7170
Condorcet-Hare	0.6578	0.6829	0.6931	0.6988	0.7017
CHUC	0.6657	0.6974	0.7105	0.7179	0.7220
MRCH	0.6641	0.6941	0.7058	0.7122	0.7156
Borda	0.6596	0.6845	0.6942	0.6998	0.7022
Range	0.6581	0.6832	0.6936	0.6990	0.7019
Approval	0.6599	0.6818	0.6904	0.6953	0.6979

All specifications above have 99 voters, 3 spatial dimensions, and 10,000 trials.

Table B3. Spatial model results with variable number of voters

	Number of voters				
	9	29	99	299	999
UEW Scores					
Plurality	0.6349	0.6273	0.6251	0.6251	0.6248
Hare	0.6377	0.6306	0.6282	0.6282	0.6278
STV	0.6377	0.6315	0.6287	0.6286	0.6283
Condorcet-Hare	0.6436	0.6349	0.6311	0.6306	0.6304
CHUC	0.6436	0.6348	0.6311	0.6306	0.6304
MRCH	0.6435	0.6348	0.6311	0.6306	0.6304
Borda	0.6447	0.6351	0.6312	0.6305	0.6304
Range	0.6462	0.6355	0.6313	0.6306	0.6304
Approval	0.6437	0.6348	0.6309	0.6304	0.6301
UEL Scores					
Plurality	0.5548	0.5574	0.5598	0.5613	0.5613
Hare	0.5638	0.5685	0.5722	0.5737	0.5736
STV	0.5738	0.5785	0.5790	0.5793	0.5795
Condorcet-Hare	0.6011	0.5947	0.5926	0.5922	0.5915
CHUC	0.5518	0.5500	0.5527	0.5535	0.5528
MRCH	0.5920	0.5827	0.5791	0.5783	0.5771
Borda	0.6037	0.5953	0.5919	0.5912	0.5901
Range	0.6088	0.5981	0.5937	0.5926	0.5916
Approval	0.5964	0.5931	0.5912	0.5910	0.5902
MU Scores					
Plurality	0.7210	0.7072	0.7026	0.7013	0.7005
Hare	0.7260	0.7132	0.7083	0.7069	0.7062
STV	0.7249	0.7104	0.7059	0.7049	0.7039
Condorcet-Hare	0.7062	0.6966	0.6931	0.6922	0.6917
CHUC	0.7338	0.7172	0.7105	0.7089	0.7079
MRCH	0.7173	0.7091	0.7058	0.7053	0.7049
Borda	0.7075	0.6982	0.6942	0.6933	0.6929
Range	0.7103	0.6984	0.6936	0.6923	0.6916
Approval	0.7000	0.6935	0.6904	0.6899	0.6892

All specifications above have 7 candidates, 3 spatial dimensions, and 10,000 trials.

Table B4. Strategic resistance of one-round versus runoff systems

Base rule	Politbarometer		Spatial	
	One-round	Runoff	One-round	Runoff
Plurality	0.7825	0.9642	0.7827	0.9572
Hare	0.9643	0.9642	0.9588	0.9572
Condorcet-Hare	0.9643	0.9668	0.9588	0.9613
Borda	0.5909	0.5645	0.5502	0.6417
Range	0.1986	0.2624	0.1045	0.1651
Approval	0.5345	0.6444	0.5407	0.6987