Selecting the runoff pair

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This version: May 9, 2019

Accepted for publication in *Public Choice* on May 27, 2019

Abstract: Although two-round voting procedures are common, the theoretical voting literature rarely discusses any such rules beyond the traditional plurality runoff rule. Therefore, using four criteria in conjunction with two data-generating processes, we define and evaluate nine "runoff pair selection rules" that comprise two rounds of voting, two candidates in the second round, and a single final winner. The four criteria are: social utility from the expected runoff winner (UEW), social utility from the expected runoff loser (UEL), representativeness of the runoff pair (Rep), and resistance to strategy (RS). We examine three rules from each of three categories: plurality rules, utilitarian rules and Condorcet rules. We find that the utilitarian rules provide relatively high UEW and UEL, but low Rep and RS. Conversely, the plurality rules provide high Rep and RS, but low UEW and UEL. Finally, the Condorcet rules provide high UEW, high RS, and a combination of UEL and Rep that depends which Condorcet rule is used.

JEL Classification Codes: D71, D72

Keywords: Runoff election, two-round system, Condorcet, Hare, Borda, approval voting, single transferable vote, CPO-STV.

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* We are grateful to those who commented on an earlier draft of this paper at the 2018 Public Choice Society conference.

1. Introduction

Voting theory is concerned primarily with evaluating rules for choosing a single winner, based on a single round of voting. Within that framework, a myriad of alternatives to plurality (or "first past the post") have been discussed over the years, such as Borda, Hare, Dodgson, Nanson, Baldwin, Black, Kemeny, Coombs, minimax, approval voting, ranked pairs, anti-plurality, beatpath and Condorcet-Hare, as well as many others.[1]

However, many elections to identify a single winner are conducted using a two-round, or runoff system. For example, of the 113 countries that hold presidential elections, 87 conduct their elections using a runoff system.[2]

While it is true that something resembling a runoff system can be implemented with a single round of voting using ballots on which voters have ranked the candidates, that is not fully equivalent to a runoff system because it does not give citizens the option of changing their reported preferences, or their decisions about whether or not to vote, between rounds. Thus, one function of a runoff is to concentrate the attention of citizens on the relative merits of the two finalists. When citizens know who the finalists are, they have stronger motivations to acquire information about them, ensuring that the eventual winner will undergo close scrutiny during the campaign and will have a majority against the one remaining alternative.

We argue that a traditional runoff system is a member of a family of two-round voting rules that is distinct from the family of one-round systems, and that the family of two-round systems invites a distinct analysis that has not been undertaken previously. This paper begins

[1] See Borda (1784), Hare (1865), Dodgson (1876), Nanson (1882), Baldwin (1926), Black (1958), Kemeny (1959), Coombs (1964), Simpson (1969), Brams and Fishburn (1978), Tideman (1987), Saari (1990), Schulze (2003) and Green-Armytage et al. (2016), respectively. Levin and Nalebuff (1995) and Tideman (2006) provide surveys. Note that what we call Hare is the single-winner version of single transferable vote, which also is known as the alternative vote, instant runoff voting and ranked choice voting. Dodgson (1876) proposed more than one election procedure, but here we mean the well-known one referred to as the "Dodgson rule" by Levin and Nalebuff (1995, pp. 24–25) and by Tideman (2006, pp. 196–199). Kemeny also is known as Kemeny-Young, because of Young (1988). Minimax also is known as Simpson, Simpson-Kramer, successive reversal, min-max, and maximin. Beatpath also is known as clone-proof Schwartz sequential dropping, or the Schulze method.

[2] International Institute for Democracy and Electoral Assistance (2018).

that analysis.

We define a "runoff pair selection rule" as follows: In the first round, voters submit ballots, which could be plurality, ranked, approval or range ballots, and those ballots are used to choose two and only two candidates, who will be considered in the second round. In the second round, voters choose between the two remaining candidates by a simple majority vote.[3]

Our goal is to evaluate such rules relative to one another. That is, *given the premise that two candidates must be chosen for a runoff*, we ask: *What is the best method for choosing them?*

To confront that question, we must first recognize that what is 'best' in the context of choosing a runoff pair might be different from what is 'best' in the context of choosing a single winner. For example, a society may wish to choose the two candidates who would, individually, give voters the most utility if elected. Alternatively, a society may wish to choose the two candidates who are most broadly representative of the electorate as a whole. When a Condorcet winner exists, the society may wish to ensure that that candidate is included in the runoff pair. The preferred rule for selecting a runoff pair may vary depending on what goals society has.

In this paper, we evaluate nine possible rules for selecting a runoff pair on the basis of four criteria. Neither the rules nor the criteria are intended to be exhaustive; rather, they are intended to provide an initial analysis and stimulate readers' imaginations as to what other analyses might be worthwhile.

The remainder of the paper is organized as follows: Section 2 defines the rules to be evaluated, Section 3 defines the criteria used to evaluate them, Section 4 describes the data used to perform our evaluations, Section 5 presents the results; Section 6 concludes. The Appendix defines additional rules and subjects them to preliminary evaluation. The Supplemental

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[3] Many runoff systems declare a winner in the first round if a single candidate receives a majority of the vote, thus canceling the second round. In the interest of simplicity, we do not treat such systems separately. In addition, some two-round systems (such as the system used to elect France's *Assemblée nationale*) sometimes field more than two candidates in the second round. To avoid complications, we do not consider such a possibility.

Appendix, which is available online, proposes additional criteria, presents results from the spatial model with different parameters, and compares some of the runoff procedures with their single-round counterparts in terms of resistance to strategy.

2. Runoff pair selection rules

2.1. Plurality rules

2.1.1. Plurality. This is the traditional rule. The runoff pair consists of the candidates with the top-two plurality scores.

2.1.2. Hare. The Hare rule eliminates candidates one by one on the basis of which candidate has the fewest first-place votes; each time a candidate is eliminated, each vote for that candidate is reassigned to the not-yet-eliminated candidate ranked next on that ballot. Application of the Hare rule chooses as the runoff pair the last two candidates who remain after the others have been eliminated.

2.1.3. Single transferable vote (STV). When used as a runoff pair selection rule, the single transferable vote (STV) rule[4] is like the Hare (last two) rule except that when a candidate receives more than a "quota" of votes, a fraction of the votes received are sent to the next candidates in those voters' rankings, so that in the end no candidate has more than a quota of votes. Because two candidates are to be chosen, a quota is one-third of the votes.[5]

2.2. Utilitarian rules

2.2.1. Borda. Each candidate's Borda score is the sum of points earned, assigned in descending

[4] Many versions of STV have been proposed; for a review, see Tideman (1995). The version we use performs fractional transfers of ballots when the first candidate achieves a quota and then divides a voter's ballot strength evenly among candidates when the voter lists them as being tied as the most preferred among non-elected, non-eliminated candidates.

[5] Thus, we use a modified "Droop quota": specifically, we use the number of voters divided by three, plus 1/1,000,000. Droop's definition of the quota named for him was 1 + the integer part of [votes ÷ (positions + 1)].

order from N – 1, where N is the number of candidates on the ballot; each position on a voter's ballot is worth the number of positions below that position. The rule selects the two candidates with the highest Borda scores.

2.2.2. Approval. Each voter gives each candidate either one point or zero points. The runoff pair consists of the two candidates with the highest point totals. In our simulations, we assume that a voter approves a candidate if and only if that candidate gives the voter utility greater than or equal to their average utility for all candidates.

2.2.3. Range. Each voter gives each candidate a score on a closed interval. The runoff pair consists of the two candidates with the greatest sums of scores. In our simulations, scores are utilities, bounded between zero and one.

2.3. Condorcet rules

2.3.1. Condorcet-Hare. For electing one candidate, the Condorcet-Hare rule is that if a Condorcet winner exists, then that candidate is selected. If not, the Hare winner is selected.[6] The Condorcet-Hare runoff pair selection rule first selects the Condorcet-Hare winner, then removes that candidate from the ballots and selects the Condorcet-Hare winner according to the revised ballots.

Many other Condorcet-efficient rules have been proposed, which likewise could be adapted for runoff pair selection. We focus on Condorcet-Hare because of evidence that it is less vulnerable to strategic manipulation than other Condorcet-efficient rules.[7] Since all Condorcet-

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[6] Other, slightly more complicated hybrid Condorcet-Hare variants can be used to ensure that the winning candidate comes from the "Smith set" (the smallest set such that everyone in the set beats everyone not in it, in head-to-head comparisons), defined in Smith (1973). Tideman (2006) provides such a rule that alternates between eliminating any candidates outside the Smith set defined with respect to the remaining candidates, and eliminating the plurality loser from votes distributed among the remaining candidates. Green-Armytage (2011) discusses four potentially attractive hybrid Condorcet-Hare rules.

[7] See, for example, Tideman (2006), Green-Armytage (2011) and Green-Armytage et al. (2016). The MRCH rule also is consistent with other research showing that the Hare rule is resistant to strategy, coupled with theorems showing that, for a broad class of voting rules, adding a provision to elect the Condorcet winner when one exists improves

efficient rules choose the same winner when a Condorcet winner exists, it is likely that other Condorcet-efficient rules would receive scores quite similar to those we calculate for Condorcet-Hare. We support such a theory in the Appendix by examining two runoff versions of the Black rule, finding that they receives similar scores to the analogous Condorcet-Hare-based rules.

2.3.2. Condorcet-Hare and utility complement (CHUC). Like range voting, this rule requires voters to score the candidates. In its first stage, the rule converts the scores into rankings and selects the Condorcet-Hare winner. In its second stage, the rule makes use of the scoring information. The utility complement score for each candidate is defined as the score difference between that candidate and the Condorcet-Hare winner, *summed over only those voters who give a lower score to the Condorcet-Hare winner*. The candidate with the highest utility complement score becomes the second member of the runoff pair.

2.3.3. Modified repeated Condorcet-Hare (MRCH). This rule begins by selecting the Condorcet-Hare winner as the first member of the runoff pair. It then proceeds by conducting a modified pairwise tally, where for each pairwise comparison between two candidates X and Y, voters who ranked the first winner above both X and Y are not taken into account. If a Condorcet winner is found according to the modified pairwise tally, that winner becomes the second member of the runoff pair. If not, a similarly modified Hare tally is conducted. In such a tally, the first winner is included as a candidate and thus can receive votes transferred from eliminated candidates (thereby preventing them from reaching later choices), but the first winner cannot be eliminated. The tally continues until the first winner and one other candidate remain; that candidate becomes the second member of the runoff pair.

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resistance to strategy. For research supporting Hare, see, for example, Chamberlin (1985), Lepelley and Mbih (1994), Lepelley and Valognes (2003), Favardin and Lepelley (2006) and Green-Armytage (2014). For the theorems supporting "Condorcification", see Green-Armytage et al. (2016) and Durand et al. (2016).

3. Criteria for evaluating runoff pair selection rules

All four of our measures belong to a broad category of evaluative criteria that are "statistical" in nature rather than "axiomatic". By that we mean that they are not logical properties that a given rule either possesses or does not possess; rather, they are statistics reflecting the frequency (and sometimes also the degree) of "success" or "failure" at achieving clearly defined goals, over a large number of simulated elections.[8]

3.1. Utility from the expected winner (UEW) is measured as the average utility that voters receive from the runoff election's winner, assuming that the set of voters and their preferences both remain unchanged between the first and second round of voting. The UEW score is a measure of the social utility associated with the candidate who is expected to win and, thus, a measure of the social utility associated with the most likely final outcome of the process.

3.2. Utility from the expected loser (UEL) is measured as the average utility that voters receive from the runoff's loser. The motivation for such a criterion is that a lower UEL score may indicate more "risk" in the process. That is, although in general we expect that the candidate who is elected in the second round will be that member of the runoff pair who is preferred by a majority as of the first round of voting, the period of time between the two rounds has the potential to introduce an unforeseen change. For example, the initial favorite might be caught in a scandal, suffer an attack of ill-health, and so on. If such an event reduces the electability of the favorite, the social utility associated with the expected runoff loser may become the social utility of the final outcome.

[8] "Axiomatic" criteria include the majority, mutual majority, Condorcet, the Condorcet loser, monotonicity, participation and consistency criteria; for definitions, see Chapter 12 of Tideman (2006). In contrast, "statistical" criteria include the frequency with which a voting rule selects the Condorcet winner, the frequency with which a voting rule is non-manipulable, the frequency with which a voting rule selects the candidate who maximizes the sum of voter utilities, the average voter utility associated with the candidate that a voting rule selects, and so on.

3.3. Representativeness (Rep) is measured as the average over voters of the higher of the utility scores assigned to the two candidates in the selected pair. The criterion is motivated by the idea that how well a voter is represented in the runoff pair can be measured by the utility that the voter receives from the candidate in the pair that the voter prefers.

3.4. Resistance to strategy (RS) is defined as the share of simulated trials in which the final winner (again assuming that voter preferences remain unchanged between rounds) is not vulnerable to strategic manipulation by any coalition of voters who collectively prefer a different candidate. In the interest of tractability, we restrict our strategic simulations to the three-candidate case.

Note that resistance to strategy is concerned only with cases wherein insincere voters can change the final outcome of the election to their mutual advantage. That is, if they can replace the non-winning member of the runoff pair only with a different non-winning candidate, we do not count that possibility as an instance of successful manipulation.[9] We assume further that in the second round, all votes will be sincere. This is to say that strategic coalitions cannot bind their members contractually to vote against their true preferences in the second round in exchange for other vote changes in the first round. Since an incentive for an insincere vote never arises in a two-candidate race, all votes will then be sincere.

4. Data

For our inquiry, we make use of two data sources: one that comes from survey data and one that is derived from a mathematical model.

As with our lists of rules and criteria, we do not intend our selection of data-generating

[9] It is not impossible that voters might engage in strategy with the goal of inserting a favored candidate in the runoff pair, even if that candidate is not expected to win. Therefore, an alternative criterion called "second-degree strategic resistance" could be defined as the share of trials in which that is impossible as well. But we do not conduct such an analysis in this paper.

processes to be exhaustive. An infinite number of data-generating processes are possible, and none of them can be said to be "definitive" in the sense of providing results that would apply to all real electorates. However, based on other research that uses data-generating processes like the one at hand,[10] we consider it likely that other reasonable data derivation processes will agree with ours largely in terms of the relative *ranking* of the runoff pair selection rules in each evaluative dimension, though not in terms of the specific *scores* that the runoff pair selection rules will receive.

4.1. Politbarometer surveys

The Politbarometer is a survey of the German public that has been conducted since 1977.[11] Respondents rate politicians on a scale from −5 to 5.[12] Our dataset includes 610 surveys, taken from 1977 to 2008.

We treat each of these 610 surveys as a hypothetical election, in which all rated politicians are candidates, and the respondents vote in accordance with their stated preferences. We map voter utilities from the [−5, 5] scale to a [0, 1] scale in a linear manner.

Respondents who assign identical ratings to all of the politicians are removed from the electorate, resulting in electorates with an average size of 595.2 voters, a minimum size of 62 voter, and a maximum size of 1,856 voters. The average number of candidates is 10.9, the minimum number is 4 and the maximum number is 21.

When testing for strategic resistance, we create separate trials from all possible three-candidate subsets from each survey, increasing the number of elections from 610 to 126,637. In each of the trials, we restrict the set of voters to those with strict preferences among all three

[10] For example, Green-Armytage (2014) and Green-Armytage et al. (2016).

[11] Ours is identical to the dataset used in Green-Armytage et al. (2016).

[12] The wording of the question can be translated from German to English as follows: "Please tell me, again with the thermometer of +5 to −5, what you think of some political leaders. +5 means that you think a lot of the politician; −5 means that you think nothing at all of him. If you do not know a politician, you naturally do not need to grade him. What do you think of …?" The data are available at http://www.gesis.org/en/elections-home/politbarometer/

candidates.

4.2. A spatial model

Our spatial model creates hypothetical elections using a three-dimensional attribute space. The location of each voter and each candidate in each attribute is determined by an independent draw from a standard normal distribution. Voter preferences over candidates are determined by proximity: The closer a candidate is to a voter, the higher is the voter's utility for that candidate.

Specifically, we first find the Euclidean distance Δ_{ij} between each voter i and each candidate j. We then translate those distances into utilities on a [0, 1] scale using the utility function $U_{ij} = e^{-\psi \Delta_{ij}^2}$, where ψ = 0.1469 is a constant that is calibrated so that the expected value of a random candidate for a random voter is approximately 1/2.

We create 100,000 simulated elections using this spatial model. Each election has 99 voters and seven candidates, excepting the three-candidate elections used to measure strategic resistance. In the Appendix, we conduct sensitivity analyses in which the number of candidates and the number of voters are varied.

4.3. Statistical significance

We use two methods to assess the statistical significance of the differences between the scores of runoff pair selection rules: the standard one and one that adjusts for first-order serial correlation, using the method recommended by Zwiers and von Storch (1995).[13]

Rather than reporting the p-value of the score difference for every pair of rules for every criterion in both data sources (which would consume an inordinate amount of space), we present the "sufficient difference for significance" (SDS) for each criterion in each data source.

[13] The latter method uses an "equivalent sample size" for the purpose of calculating the standard error, equal to $(1-\rho)/(1+\rho)$, where ρ is the serial correlation coefficient for the difference in two rules' scores in successive trials. The formula is given in equation 6 of Zwiers and von Storch (1995), with the correction of a typo in the original paper that displays incorrectly a "ρ" as a "τ."

The SDS is the value such that if the difference between the score of two rules exceeds it, that difference must be significant at the 5% level or better.

In our tables, "SDS-I" and "SDS-II" denote SDS values for the standard method and the alternative method described above, respectively.

5. Results

In Section 5.1, we present the scores themselves in brief. In Section 5.2, we discuss the intuition behind the scores.

5.1. Scores

Table 1 displays the results derived from the Politbarometer data, and Table 2 displays the results derived from the spatial model. Each of the tables reports the scores of the nine rules by the four criteria, to four decimal places. Table 1 displays both SDS-I and SDS-II values of statistical significance, while Table 2 displays only SDS-I values, because the trials in the spatial model are independent, so serial correlation is not a concern.

Figures 1 and 2 provide visualizations of the information shown in Tables 1 and 2. Each figure displays two of the four criteria as two dimensions of a scatter plot depicting the relative performances of the nine rules. Figure 1 displays resistance to strategy (RS) on the horizontal axis and utility from the expected winner (UEW) on the vertical axis; panel 1a does the same for the Politbarometer data and panel 1b for the spatial model. Figure 2 displays representativeness (Rep) on the horizontal axis and utility from the expected loser (UEL) on the vertical axis; again, the two panels correspond to the two data sources.

The figures are drawn using scales chosen to show the differences among the rules, leading to vastly different scales from one axis to another. Thus, the scale for resistance to strategy goes in both cases from 0 to 1, while the scale for utility from the expected winner goes in one case from 0.7286 to 0.7304 and in the other case from 0.625 to 0.633. What the differences mean for the relative values of rules is hard to say because we have no experience with such scales. Nevertheless, the tradeoff between utility of the expected winner and resistance to strategy implies that an electorate that cared only about utility of the expected winner under sincere voting would select a utilitarian rule, while an electorate that cared at all about resistance to strategy would reject all of the utilitarian rules. While the differences in utility from the expected winner may seem very small, they are not necessarily inconsequential. Different rules may select the same winner in the overwhelming majority of cases and then choose winners with very different utilities in a few cases. Any electorate seeking to use the results to evaluate possible rules would need to deal with them for long enough to get an understanding of what the differences in values mean.

The two panels of Figure 1 agree on several salient results. In both panels, the utilitarian rules occupy the upper left of the graph, indicating that they likely will lead to relatively high social welfare if votes are sincere, but they also are more likely to provide incentives for insincere voting. Range voting is most likely to be vulnerable to strategy, followed by Borda, then approval.

The plurality-based rules occupy the lower rights of both panels, indicating that they are relatively unlikely to be manipulable, but their expected winners provide comparatively low social welfare. The traditional plurality runoff system offers the lowest UEW score, followed by Hare, then STV.

The Condorcet rules are near the upper rights of both panels, indicating that the expected result with sincere voting tends to offer relatively high social welfare (though in most cases incrementally less than the utilitarian rules), and the outcomes are unlikely to be vulnerable to strategic manipulation.

The two panels of Figure 2 likewise are largely in agreement with each other. In both panels, the utilitarian rules once again occupy the upper lefts of the graphs. Tha finding indicates that social welfare likely still will be relatively high if the runoff produces an unexpected result, but that a relatively large number of voters likely will feel that neither member of the runoff pair represents them well.

The plurality-based rules are below and to the right of the utilitarian rules, indicating that they allow more voters to feel represented by at least one member of the runoff pair, but that an unexpected runoff result could lead to a greater drop in social welfare. Of the three plurality-based rules, the traditional rule has both the lowest UEL scores and the lowest representativeness scores. STV has the highest UEL scores in this category, while Hare has the highest representativeness scores.

Unlike Figure 1, which shows the three Condorcet rules clustered closely together, Figure 2 illustrates strong contrasts among them. In Figure 2, Condorcet-Hare is in the same region of the graph as the utilitarian rules, offering high UEL, but low representativeness. CHUC, on the other hand, is at the far end of the other side of the spectrum, receiving both the highest representativeness score and the lowest UEL score of all nine rules. Finally, MRCH is nearer the middle of the representativeness-UEL frontier. In one of the few notable differences between the two panels of Figure 2, MRCH has scores similar to STV according to the spatial model data, but lies below and to the right of all three plurality rules according to the Politbarometer data.

Table 1. Results from the Politbarometer data

	UEW	UEL	Rep	RS
Plurality-based rules				
Plurality	0.7288	0.6484	0.8122	0.9694
Hare	0.7290	0.6503	0.8158	0.9694
STV	0.7293	0.6607	0.8131	0.9694
Condorcet rules				
Condorcet-Hare	0.7300	0.6714	0.8028	0.9726
CHUC	0.7300	0.6342	0.8174	0.9694
MRCH	0.7300	0.6450	0.8165	0.9694
Utilitarian rules				
Borda	0.7301	0.6723	0.8034	0.5426
Range	0.7302	0.6734	0.8034	0.2526
Approval	0.7301	0.6715	0.8014	0.6408
Statistical significance				
SDS-I	0.0006	0.0044	0.0018	0.0141
SDS-II	0.0007	0.0075	0.0028	0.0351

Table 2. Results from the spatial model

	UEW	UEL	Rep	RS
Plurality-based rules				
Plurality	0.6257	0.5608	0.7030	0.9572
Hare	0.6288	0.5730	0.7087	0.9572
STV	0.6293	0.5794	0.7065	0.9599
Condorcet rules				
Condorcet-Hare	0.6318	0.5932	0.6937	0.9613
CHUC	0.6318	0.5535	0.7111	0.9572
MRCH	0.6318	0.5796	0.7065	0.9572
Utilitarian rules				
Borda	0.6319	0.5925	0.6947	0.6417
Range	0.6320	0.5943	0.6940	0.1651
Approval	0.6317	0.5919	0.6909	0.6987
Statistical significance				
SDS-I	0.0001	0.0003	0.0002	0.0031

<u>Abbreviations</u>

UEW = utility from the expected winner
UEL = utility from the expected loser
Rep = representativeness
RS = resistance to Strategy
STV = single transferable vote
CHUC = Condorcet-Hare & utility complement
MRCH = modified repeated Condorcet-Hare
SDS = sufficient difference for significance

Figure 1. Resistance to strategy and utility from the expected winner

1a. Politbarometer data

1b. Spatial model

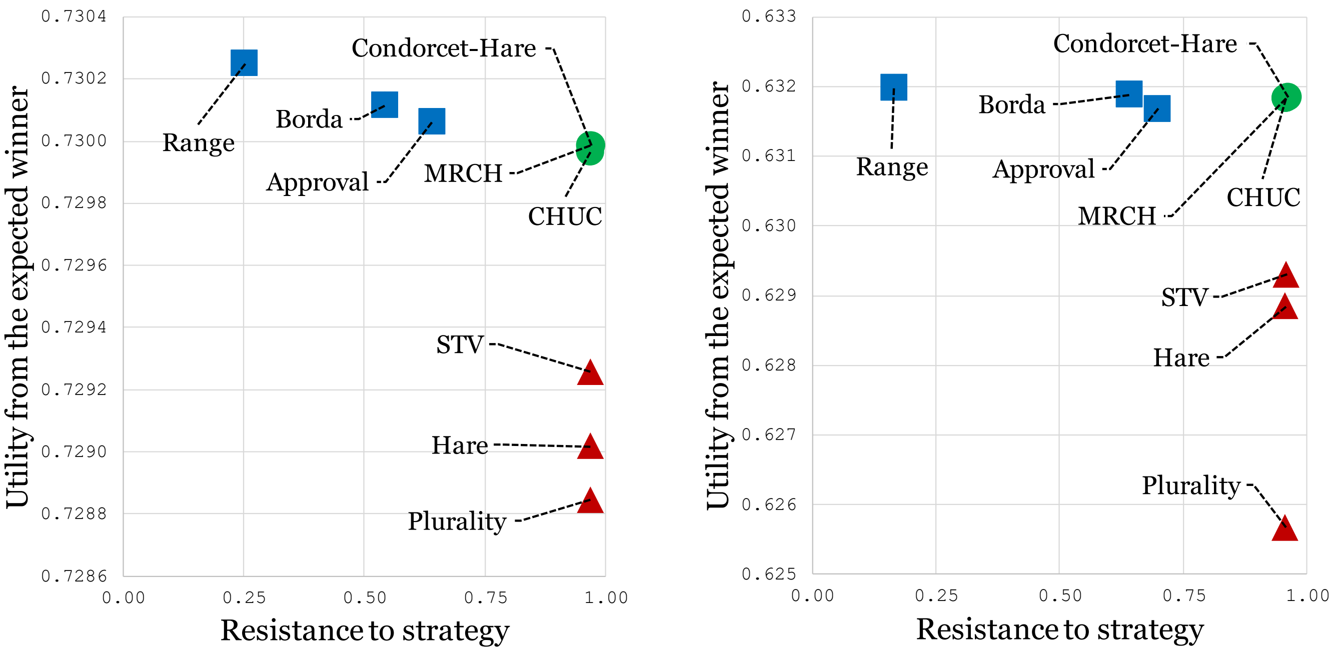
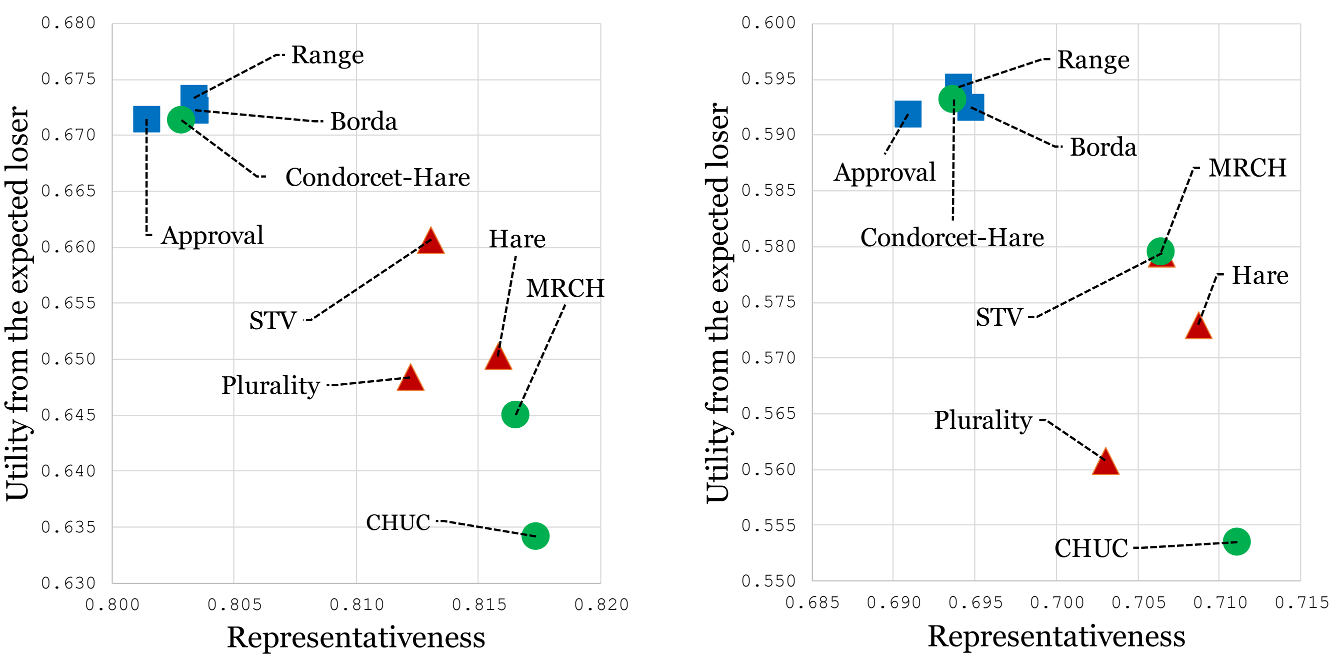


Figure 2. Representativeness and utility from the expected loser

2a. Politbarometer data

2b. Spatial model



<u>Key</u>

■ = utilitarian rule
▲ = plurality rule
● = Condorcet rule

<u>Abbreviations</u>

STV = single transferable vote
CHUC = Condorcet-Hare and utility complement
MRCH = modified repeated Condorcet-Hare

Note: The axes are scaled to emphasize differences between rules, and are therefore non-uniform.

5.2. Discussion

The utilitarian rules have high UEW and UEL scores because they tend to select the two candidates who deliver the highest sums of voter utilities. Range voting does that explicitly, while Borda and approval can be said to approximate the same results with ranked ballots and approval ballots, respectively. The high-utility candidates are likely to be in the center of the political distribution, so they are unlikely to appeal to voters who are far from the center; thus, the utilitarian rules receive low representativeness scores.

In the established literature on one-round single winner rules, it often has been found that rules like Borda, range and approval are highly manipulable.[14] Here, we extend that result to the two-round case.

On the other hand, the same literature generally has found that Hare is relatively unlikely to be subject to strategic manipulation.[15] In the three-candidate case that we use for our strategy simulations herein, the plurality runoff and Hare runoff systems are equivalent, but as the number of candidates increases, their RS scores should diverge in favor of Hare.[16]

Plurality receives moderate representativeness scores, higher than any of the utilitarian methods. To understand that finding, note that the first round of plurality rule is equivalent to single non-transferable vote (SNTV) for electing two candidates, which traditionally is classified as a semi-proportional election rule.[17] Since STV is a fully proportional system, it is intuitive that it receives higher representativeness scores than plurality does. However, Hare receives higher scores still by our definition of representativeness. That is because STV's use of transferred surplus votes allows voters who already are satisfied with the first member of the pair to have

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[14] For example, Chamberlin (1985), Lepelley and Valognes (2003) and Favardin and Lepelley (2006) report that Borda often is vulnerable to coalitional manipulation. Tideman (2006), Green-Armytage (2014) and Green-Armytage et al. (2016) extend that result to include range and approval voting.

[15] The result is found in each of the sources cited directly above.

[16] Established directly in Green-Armytage (2014).

[17] For example, by Lakeman (1974).

further say in choosing the second member of the pair, a provision that increases UEW and UEL, but reduces representativeness.

All three Condorcet rules have relatively favorable combinations of UEW scores and RS scores.[18] Intuitively, when a Condorcet winner exists, that candidate is often (though not always) one who provides maximum or close-to-maximum social welfare.[19] However, whereas sincere voting under utilitarian methods can produce non-majoritarian outcomes that can be overturned by majorities who vote strategically, Condorcet winners face no majority opposition. Furthermore, to unseat a Condorcet winner under a Condorcet-efficient rule, strategists must first create an artificial majority rule cycle and then make their preferred candidate the winner according to the applicable completion rule. We choose Hare as the completion rule for our Condorcet method to take advantage of its anti-strategic properties.

The three Condorcet rules differ substantially in terms of representativeness and UEL. At one end of the spectrum, our implementation of Condorcet-Hare repeats the same counting procedure for the selection of both members of the runoff pair, so that both are likely to come from the political center. That likelihood does little to add representation for voters who are unsatisfied with the first member of the pair, but it means that the risk of electing a candidate outside the mainstream is minimal.

At the other end of the spectrum, by selecting the second candidate based on the sum of positive differences in ratings versus the first candidate, CHUC tends to select a candidate who represents the most passionate opposition to the likely winner. Thus, if the first member of the pair is near the political center, the second member is likely far from the center, and thus may provide low social welfare in the event of an unexpected second-round outcome.

Finally, MRCH takes an intermediate approach between repeated Condorcet-Hare and

[18] The result parallels Green-Armytage et al. (2016), who found that the one-round version of Condorcet-Hare has a favorable combination of resistance to strategy and utilitarian efficiency.

[19] That always would be true in a spatial model with utility determined by distance and a distribution of voter locations with point symmetry.

CHUC. That is, although MRCH does omit the voters closest to the first member of the pair when deciding on the second, it does not give added favor to candidates who are further from the first winner, thus provoking greater rating differences.

6. Conclusion

Of the nine rules we evaluate, the traditional plurality runoff system offers the lowest social welfare when the expected runoff winner prevails. It is relatively resistant to strategic voting in the three-candidate case, but that resistance is likely to diminish as the number of candidates rises. Its first round is equivalent to the two-position case of the semi-proportional SNTV system and, as such, it is modestly representative. Thus, the plurality runoff rule might be most attractive to societies that place primary value on simplicity and familiarity, and secondary value on representativeness and resistance to strategy.

Runoff systems based on Hare and STV can improve modestly over plurality in terms of both expected social welfare and representativeness. Although STV is appropriate if *proportional* representation is the goal, the runoff pair under Hare will more often include a second candidate that appeals strongly to voters not satisfied with the first candidate. Such systems might be attractive to a society that places primary value on representativeness and resistance to strategy, and secondary value on expected social welfare.

The utilitarian rules (viz. Borda, range, and approval voting) generate higher social welfare than the plurality-based rules, both when the runoff election goes as predicted and when it does not. However, those rules are vulnerable to strategy, and they may generate runoff pairs comprised of two similar candidates. Thus, the utilitarian rules might be most attractive to a society that places primary value on expected social welfare, trusts its members not to use strategy, and places little value on having strong contrasts between the election’s finalists.

The Condorcet rules we have evaluated are Condorcet-Hare, Condorcet Hare and utility complement (CHUC), and modified repeated Condorcet Hare (MRCH). Any of them might be attractive to a societies whose primary values are to maximize the utility from the expected runoff winner and to minimize the likelihood that strategic voting will be successful.

As to which of the three Condorcet rules such a society might prefer, the answer depends on the society's secondary values. If it places little value on the two members of the runoff pair having strongly diverging points of view but places high value on minimizing the loss of social welfare associated with a second-round upset, it might be most attracted to the repeated Condorcet-Hare rule.

On the other hand, if the society had opposite priorities regarding representativeness and the utility from the expected runoff loser, it might be most attracted to the CHUC rule.

Finally, if the society wished to pursue some mixture of these two competing goals, it might be most attracted to the MRCH rule.

Acknowledgements

We are grateful to those who commented on an earlier draft of this paper at the 2018 Public Choice Society meetings.

Appendix: Additional rules to consider

This paper aims not only to evaluate nine run-off pair selection rules in particular, but also to encourage a broader discussion of what other rules might be worth evaluating. We consider that project important because we have not established that the nine rules covered above are superior to all other runoff pair selection rules and, thus, it is quite possible that a society may find another rule to be preferable. Therefore, we continue by defining an additional 14 rules, bringing our total from nine to 23. We then evaluate them by the social utility from the expected runoff winner (UEW), social utility from the expected runoff loser (UEL), and representativeness of the runoff pair (Rep) criteria. (We omit the resistance to strategy criterion here because it requires rule-specific algorithms that we have devised for use only in our core analysis.) For brevity, we only present results from the Politbarometer data.

A.1. Utility complement rules

These are relatives of the Condorcet-Hare and utility complement (CHUC) rule.

A.1.1. Range and utility complement (RUC) is like CHUC except that range voting is used rather than Condorcet-Hare to select the first member of the pair.

A.1.2. Condorcet-Hare and Borda complement (CHBC) is like CHUC except that the "Borda complement" is chosen as the second member of the pair instead of the utility complement. The Borda complement is analogous to the utility complement, but uses rankings instead of ratings and substitutes ranking differentials for rating differentials.

A.1.3. Borda and Borda complement (BBC) first selects the Borda winner and then selects the Borda complement.

A.2. Repeated rules

These select the first member of the pair according to a given base rule. Then they delete the winning candidate from the ballots and select the second member of the pair by applying the same base rule to the modified ballots. The Condorcet-Hare rule we evaluate in our main analysis is an example of a repeated rule. Also, repeated versions of range and approval voting would be equivalent to the versions we evaluate above.

A.2.1. Repeated plurality uses plurality as its base rule, but must nevertheless have ranked ballots in order to perform the second iteration.

A.2.2. Repeated Hare uses Hare as its base rule.

A.2.3. Repeated Borda uses Borda as its base rule.

A.2.4. Repeated Black uses the base rule that selects the Condorcet winner when one exists, and selects the Borda winner otherwise.

A.3. Modified repeated rules

These rules are relatives of modified repeated Condorcet-Hare (MRCH). They first select the winner according to a given base rule, then select the winner according to the same base rule, neglecting the preferences of voters who are satisfied with the candidate selected first. Two otherwise-obvious rules, MR Borda and MR Hare, are unnecessary to include in our review because they turn out to be mathematically equivalent to rules already defined above. That is, MR Borda is equivalent to Borda BC, and MR Hare is equivalent to our main implementation of Hare.

A.3.1. Modified repeated range first selects the range voting winner. Then each ballot is modified so that ratings not exceeding the rating of the first selection are set to zero, and the candidate with the largest sum of ratings according to the modified ballots is selected as the second member of the pair.

A.3.2. Modified repeated approval first selects the approval voting winner. Then the ballots cast by the voters who approved of that candidate are removed from the count, and the approval winner according to the remaining ballots is selected as the second member of the pair.

A.4. Single transferable vote rules

Each of the following rules uses some combination of STV and Condorcet's method of pairwise comparison.

A.4.1. Comparison of pairs of outcomes by single transferable vote (CPO-STV) is conducted as defined in Tideman (1995), with two slots to be filled and a "minimax" completion method to be used in the case that no two-candidate set is dominant.

A.4.2. Condorcet-Hare-constrained CPO-STV (CHC-CPO-STV) conducts a restricted CPO-STV tally, with only the outcomes containing the Condorcet-Hare winner considered.

A.4.3. Condorcet-Hare-constrained STV (CHC-STV) selects the Condorcet-Hare winner and then conducts a modified two-slot STV tally in which the Condorcet-Hare winner can fractionally transfer surplus votes, but cannot be eliminated.

A.5. Miscellaneous rules

A.5.1. Condorcet-Hare and closest competitor (CHCC) selects the Condorcet-Hare winner and the candidate who receives the most votes in pairwise comparison with the Condorcet-Hare winner.

A.5.2. Normalized range normalizes each range ballot such that the highest score is equal to one, the lowest score is equal to zero and all scores in between are mapped to the [0, 1] interval in a linear manner. The two candidates with the largest sums of scores are selected for the runoff.

A.5.3. Aside on multi-round systems. Another possible extension of the "runoff pair selection rule" concept would be to conduct a multi-round voting procedure, with the runoff as the last round, the selection of the runoff pair in the penultimate round, and any prior reductions in the number of candidates in preceding rounds. For example, a system where the first round reduces the field of candidates to nine using a multi-winner version of CHC-STV, the second round reduces the field to two using CHC-STV as defined above, and the last round is the final runoff. We invite future discussion of this topic, but provide no formal analysis of it here.

A.6. Results including additional rules

Table A1 presents the UEW, UEL and Rep scores of 23 runoff pair selection rules. The rules are ranked separately by each criterion. Here, we will comment on a few salient outcomes.

First, the repeated rules have relatively low scores by the “representativeness” criterion. That is intuitive: Since the rules perform the same algorithm twice, they are likely to pick similar candidates for the two members of the runoff pair. Thus, voters who are dissatisfied with the first candidate likely will be dissatisfied with the second candidate as well.

Second, the new utility complement rules, namely RUC, Borda BC and CHBC, receive scores that are more similar to CHUC than to the utilitarian rules discussed in the main analysis. This is an example of the principle that a runoff pair selection rule’s properties are not determined solely by its base rule.

Third, the CHC-CPO-STV and CHC-STV rules seem to be “well-rounded”, in that they receive scores that are above average in all three dimensions. A possible drawback to these rules is their relative complexity; CHC-CPO-STV performs marginally better than CHC-STV by most criteria, but CHC-STV arguably is the easier of the two to comprehend.

Table A1. Politbarometer results with additional rules

Rank	UEW		UEL		Rep	
	Rule	Score	Rule	Score	Rule	Score
1	MR range	0.7303	range	0.6734	CHUC	0.8174
2	range	0.7302	norm range	0.6729	RUC	0.8169
3	RUC	0.7301	R Borda	0.6725	CHBC	0.8169
4	norm range	0.7301	Borda	0.6723	BBC	0.8167
5	Borda	0.7301	R Black	0.6715	MRCH	0.8166
6	R Borda	0.7301	approval	0.6715	Hare	0.8158
7	R Black	0.7301	R Condorcet-Hare	0.6714	STV	0.8131
8	approval	0.7301	R Hare	0.6693	CPO-STV	0.8128
9	BBC	0.7301	CHC-CPO-STV	0.6633	CHC-CPO-STV	0.8123
10	CHCC	0.7300	CPO-STV	0.6631	plurality	0.8122
11	R Condorcet-Hare	0.7300	CHCC	0.6622	CHC-STV	0.8121
12	MRCH	0.7300	CHC-STV	0.6620	MR approval	0.8120
13	CHC-CPO-STV	0.7300	STV	0.6607	MR range	0.8109
14	CHC-STV	0.7300	R plurality	0.6597	CHCC	0.8095
15	CHUC	0.7300	Hare	0.6503	Borda	0.8034
16	CHBC	0.7300	plurality	0.6484	range	0.8034
17	R Hare	0.7298	MRCH	0.6449	norm range	0.8033
18	MR approval	0.7297	BBC	0.6389	R Hare	0.8029
19	CPO-STV	0.7293	CHBC	0.6387	R Condorcet-Hare	0.8028
20	STV	0.7293	MR range	0.6375	R Black	0.8028
21	Hare	0.7290	RUC	0.6356	R Borda	0.8027
22	plurality	0.7288	CHUC	0.6342	approval	0.8014
23	R plurality	0.7286	MR approval	0.6272	R plurality	0.8006

Abbreviations

UEW = utility from the runoff Winner
UEL = utility from the runoff loser
Rep = representatitiveness
R = repeated
MR = modified repeated
CH = Condorcet-Hare
CHC = CH-constrained
CPO = comparison of pairs of outcomes
STV = single transferable vote
UC = utility complement
BC = Borda complement
CC = closest competitor
norm = normalized

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