## FINAL EXAM, ECON 4044, SPRING 2012 NAME:

Fill in the blanks, and/or answer in the space provided, boxing your final answers. You must show correct work for credit.

**1. Optimal government size.** Suppose that there is a private good, with a market that is efficient in the absence of taxation, and a public good, that can only be provided by the government, using revenue from a per-unit tax of  $\tau$  on the private good. Let the marginal benefit and cost for the private good (with quantity *x*) be as follows:

$$MB(x) = 100 - 3x$$
  $MC(x) = 20 + x$ 

Let the marginal benefit and cost of the public good (with quantity y) be as follows:

MB(y) = 75 - 3y MC(y) = 20

At the optimum government size, the shadow value of government revenue and the shadow cost of government expenditure are both 3/2. So, if this value is represented by  $1 + \lambda$ , then  $\lambda = 1/2$ . **1-1.** Find the tax revenue function  $R(\tau)$ , and the deadweight loss function  $DWL(\tau)$ .

**1-2.** Find the optimal per-unit tax  $\tau^{o}$ , and the resulting revenue  $R(\tau^{o})$ .

**1-3.** Find the optimal quantity of the public good  $y^o$ , and the resulting expenditure  $E(y^o)$ .

**2. Election algorithms.** Suppose that there are three candidates in an election, named A, B, C, and D. There are 100 voters in total, who vote as follows:

15: $A > B > C > D$	$24: B \succ C \succ A \succ D$	21: C > A > B > D
20: D > A > B > C	10: D > B > C > A	10: D > C > A > B

**2-1. Plurality.** The plurality winner is \_\_\_\_\_. Show your work, including scores for each candidate.

**2-2. Instant runoff voting (IRV).** The IRV winner is \_\_\_\_\_. Use the table below to show the vote tallies in each round.

	А	В	С	D
round 1				
round 2				
round 3				

**2-3. Minimax.** The minimax winner is \_\_\_\_\_. Construct the pairwise matrix below-left to find the candidates' scores, and construct a tournament diagram below-right, by drawing arrows from winning candidates to defeated candidates.



**2-4. Borda count.** The Borda winner is \_\_\_\_\_. Show your work, including scores for each candidate.

**2-5. Ranked pairs.** The ranked pairs winner is \_\_\_\_\_. Which defeats are ignored?

**2-6. Benham.** The Benham winner is \_\_\_\_\_. Show your work.

**2-7. Condorcet winner, majority rule cycle.** What is the definition of a Condorcet winner? Is there a Condorcet winner in this election? If so, who is it? Is there a majority rule cycle in this election? If so, describe it.

3. Public good decision processes. Suppose that, in a certain town with only five people, the park is a non-rival and non-excludable good. The utility functions for each citizen *i* can be represented as  $U_i = x_i + \alpha_i \ln y$ , where *y* is the amount of money that the town spends to build its park, and  $x_i$  is the amount of money that person *i* has left over for private consumption. The  $\alpha_i$  values are as follows:  $\alpha_1 = 20$ ,  $\alpha_2 = 21$ ,  $\alpha_3 = 23$ ,  $\alpha_4 = 26$ , and  $\alpha_5 = 30$ .

**3-1.** What is the Pareto efficient expenditure on the park, *y<sup>o</sup>*?

**3-2.** If the citizens of this town are absolutely incapable of coordination and bargaining, what is the Nash equilibrium expenditure on the park?

**3-3.** If the citizens of the town agree to divide the cost of the park evenly among them, and then decide how much to spend on the park using a process of iterative majority rule voting, what value of *y* will be an equilibrium in this voting process?

**3-4.** An amazing psychic visits the town, and makes the citizens' utility functions known to each other. Armed with this knowledge, they decide to implement a Lindahl tax scheme. Thus, they decide that the fractional park cost shares will be  $s_1 =$ \_\_\_\_\_,  $s_2 =$ \_\_\_\_\_,  $s_3 =$ \_\_\_\_\_\_,  $s_4 =$ \_\_\_\_\_\_, and  $s_5 =$ \_\_\_\_\_. Given these tax shares, the majority voting equilibrium will be y =\_\_\_\_\_.

**4. Positive externality.** Suppose the market for a certain good (e.g. 'education') is perfectly competitive, but that the good causes a *positive* externality. Marginal private benefit, marginal external benefit, and marginal cost are given by the functions below:

$$MB = 200 - \frac{1}{25}Q$$
  $MEB = 110$   $MC = 60 + \frac{1}{100}Q$ 

**4-1. No policy.** Given that there is no policy to address the externality, find the equilibrium quantity, price, consumer surplus, producer surplus, external benefit, and total economic surplus.  $Q^* = \_\_\_\_P^* = \_\_\_CS^* = \_\_\_PS^* = \_\_PS^* = \_\_EB^* = \_\_TES^* = \_\_\_PS^* = \_\_PS^* = \_PS^* = \_\_PS^* = \_\_PS^* = \_PS^* = \_\_PS^* = \_\_PS^* = \_\_PS^* = \_PS^* = \_\_PS^* = \_PS^* = \_\_PS^* = \_PS^* = \_\_PS^* =\_\_PS^* = \_\_PS^* = \_\_PS^* = \_\_PS^* = \_\_PS^* = \_\_PS^* = \_\_PS^*$ 

Graph the market with no policy intervention, labeling *CS*<sup>\*</sup>, *PS*<sup>\*</sup>, and deadweight loss (*DWL*).

4-2. Pigovian subsidy. To maximize total economic surplus, the government should offer a subsidy of  $\sigma^o =$  \_\_\_\_\_ per unit to the consumers. Given this, find the equilibrium quantity, price, consumer surplus, producer surplus, external benefit, government expenditure, and total economic surplus.

 $Q^{o} = \underline{\qquad} P^{o} = \underline{\qquad} CS^{o} = \underline{\qquad} PS^{o} = \underline{\qquad} CS^{o} = \underline{\qquad} CS$ 

**4-3. Comparison.** Who is made better off by the subsidy policy? Who is made worse off? What is the overall change in total economic surplus?

Parties made better off (by what amount?):

Parties made worse off (by what amount?):

Change in *TES* (increase or decrease?):

## 5. Rawlsian, leaky-bucket redistribution

Suppose that you are with others behind a 'veil of ignorance', deciding how much redistribution there should be in your society. Once the decision is made, there is probability  $\pi_P$ that you will become a poor person, with pre-redistribution wealth  $w_P$ , and there is probability  $\pi_R$  that you will become a rich person, with pre-redistribution wealth  $w_R$ . Unfortunately, redistribution is 'leaky'; that is, for every \$1 that is taxed away from a rich person, only \$ $\delta$ reaches the poor, where  $\delta \in (0, 1)$ .

5-1. General budget constraint. If  $c_P$  is the post-redistribution consumption of a poor person,  $c_R$  is the post-redistribution consumption of a rich person, and x is the amount of money taxed away from each rich person, find the budget constraint in the form  $\_c_P + \_c_R = \_w_P + \_w_R$ 

**5-2. General first order condition.** Given that your Von Neumann-Morgenstern utility function is U(c), with U'(c) > 0 and U''(c) < 0, find the first order condition for expected utility maximization, which will hold when you get precisely the amount of redistribution you want.

5-3. Numerical example. Suppose that  $\pi_P = 9/10$ ,  $\pi_R = 1/10$ ,  $w_P = 12,000$ ,  $w_R = 180,000$ ,  $\delta = 1/2$  and  $U(c) = \sqrt{c}$ . Find the values of  $c_P$ ,  $c_R$ , and x that result from the amount of redistribution that you would ideally like there to be, given your utility function. In your calculations, put a box around the final versions of the budget constraint and the first order condition, as well as the ideal values of  $c_P$ ,  $c_R$ , and x.

**6. Proportional representation, part 1.** When electing a legislature, what is the difference between using a proportional representation system, and a single-member district system such as plurality?

**7. Proportional representation, part 2.** Name two countries that use a proportional representation system to elect their national legislature (or lower house, if there are more than one), and two countries that use a single-member district method. If you know specifically what kind of system each country uses, you should write this as well.

**8. Tiebout model, part 1.** In Tiebout's model, what are the benefits of having a large number of independent local jurisdictions that make independent decisions regarding the public goods they provide, and the taxes that they charge to residents?

**9. Tiebout model, part 2.** What are the major limiting assumptions of Tiebout's model? Describe the general types of situations in which devolution of responsibilities to more local levels of government can be problematic, and thus decision-making at higher levels of government is likely to be preferable.

**10. Rawlsian redistribution -- intuition.** Explain clearly how the Rawlsian framework uses risk aversion and decreasing marginal utility of wealth to provide an argument for redistribution of wealth, even if this redistribution results in a loss of efficiency.

**11. Single transferable vote.** Describe how the single transferable vote election system works, in as much detail as you can. How does it differ from a party list system?