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Determining the progressivity / regressivity of simple tax functions

Preliminary definitions

Let Y_i be person i's taxable income, or wealth, or inheritance, etc. (depending on the example). Let T_i be the tax paid by person i, and let $t_i \equiv T_i/Y_i$ be person i's average tax rate. Here, we are interested in whether each tax itself is progressive or regressive, and whether changes to the parameters of each tax will make it more or less progressive.

$\Omega \equiv t_2 - t_1$ will be used as a measure of progressivity, defined as the comparison of two individuals' average tax rates. By assumption, $Y_2 > Y_1$, so positive values of Ω indicate progressivity, and higher values of Ω indicate greater progressivity.

1. Linear tax with nonzero intercept

Suppose that each person i pays a tax of κ no matter what if κ if positive, or receives a credit of $|\kappa|$ if κ is negative. Suppose also that each person pays a constant marginal rate m on his or her income.

$$T_i = \kappa + mY_i$$

$$t_i = \frac{\kappa + mY_i}{Y_i} = \frac{\kappa}{Y_i} + m$$

$$\frac{\partial t_i}{\partial Y_i} = -\frac{\kappa}{Y_i^2}$$

If $\alpha > 0$, $\partial t_i/\partial Y_i < 0$ (that is, average tax rate decreases when income increases), so the tax is regressive. If $\kappa < 0$, $\partial t_i/\partial Y_i > 0$, so the tax is progressive. In other words, a flat tax plus a constant tax is regressive, but a flat tax minus a constant credit is progressive.

We use the following calculations to determine the impact of κ and m on the tax's progressivity or regressivity.

$$\Omega \equiv t_2 - t_1 = \left(\frac{\kappa}{Y_2} + m\right) - \left(\frac{\kappa}{Y_1} + m\right)$$

$$\Omega = \kappa\left(\frac{1}{Y_2} - \frac{1}{Y_1}\right)$$

$$\frac{\partial \Omega}{\partial \kappa} = \left(\frac{1}{Y_2} - \frac{1}{Y_1}\right) < 0 \qquad\qquad \frac{\partial \Omega}{\partial m} = 0$$

What we have found is that lower values of κ (i.e. greater credits) increase progressivity, whereas changes in the marginal rate don't affect progressivity.

2. Estate tax-like structure: two brackets, with rates 0 and m

Suppose that each person i pays no tax on inheritance up to a certain exemption amount E, and then pays a constant marginal rate m on all inheritance exceeding E, if their inheritance is greater than E.

$$T_i = \max\{0, \qquad m(Y_i - E)\}$$

If $Y_i < E$, $T_i = 0$, $t_i = 0$, and $\partial t_i / \partial Y_i = 0$. In other words, below the exemption, the tax is neither progressive nor regressive, because there is no tax. So, we proceed to the case in which $Y_i > 0$:

$$t_i = \frac{m(Y_i - E)}{Y_i} = m - \frac{mE}{Y_i}$$

$$\frac{\partial t_i}{\partial Y_i} = \frac{mE}{Y_i^2} > 0$$

Thus, because $\partial t_i / \partial Y_i > 0, \forall Y_i$, this type of tax structure is necessarily progressive.

In order to evaluate the impact of changes in m and E, we need to divide the analysis into two possible cases. The first is the case in which both people have inheritances above the exemption, and the second is the case in which only one person is above the exemption. There is also a third case, in which neither person is above the exemption; however, this case is trivial, as both people pay a tax of zero, and thus marginal changes in the parameters won't have any effect.

Estate tax, case 1: $Y_2 > E, Y_1 > E$

$$\Omega \equiv t_2 - t_1 = \left(m - \frac{mE}{Y_2}\right) - \left(m - \frac{mE}{Y_1}\right)$$

$$\Omega = \frac{mE}{Y_1} - \frac{mE}{Y_2} = mE\left(\frac{1}{Y_1} - \frac{1}{Y_2}\right) > 0$$

$$\frac{\partial \Omega}{\partial m} = E\left(\frac{1}{Y_1} - \frac{1}{Y_2}\right) > 0 \qquad\qquad \frac{\partial \Omega}{\partial E} = m\left(\frac{1}{Y_1} - \frac{1}{Y_2}\right) > 0$$

Estate tax, case 2: $Y_2 > E, Y_1 < E$

$$\Omega = \left(m - \frac{mE}{Y_2}\right) - 0$$

$$\frac{\partial \Omega}{\partial m} = 1 - \frac{E}{Y_2} > 0 \qquad \frac{\partial \Omega}{\partial E} = -\frac{m}{Y_2} < 0$$

What we have found is as follows. First, increases in the marginal rate m unambiguously increase the progressivity of the tax. Second, the effect of increasing the exemption E is more ambiguous. That is, when we are comparing two people on either side of the exemption, increasing the exemption reduces progressivity, but when we are comparing two people above the exemption, increasing the exemption increases productivity. In the case of the U.S. estate tax, since most people are in fact below the exemption, it may be argued that the former comparison is more significant.

3. Social security tax-like structure: two brackets, with rates m and 0

$$T_i = \min\{mC, \qquad mY_i\}$$

If $Y_i < C$, $T_i = mY_i$, and $t_i = m$, so it's proportional. If $Y_i > C$,

$$t_i = \frac{mC}{Y_i}$$

$$\frac{\partial t_i}{\partial Y_i} = -\frac{mC}{Y_i^2} < 0$$

Thus, it's regressive, if anything.

If $Y_2 > C$, and $Y_1 > C$,

$$\Omega = \frac{mC}{Y_2} - \frac{mC}{Y_1}$$

$$\Omega = mC\left(\frac{1}{Y_2} - \frac{1}{Y_1}\right)$$

$$\frac{\partial \Omega}{\partial m} = C\left(\frac{1}{Y_2} - \frac{1}{Y_1}\right) < 0 \qquad \frac{\partial \Omega}{\partial C} = m\left(\frac{1}{Y_2} - \frac{1}{Y_1}\right) < 0$$

If $Y_2 > C$ and $Y_1 < C$

$$\Omega = \frac{mC}{Y_2} - m$$

$$\frac{\partial \Omega}{\partial m} = \frac{C}{Y_2} - 1 < 0 \qquad \frac{\partial \Omega}{\partial C} = \frac{m}{Y_2} > 0$$

What we’ve found is as follows. Raising the marginal rate makes things more regressive, if anything. Comparing two people on either side of the cap, raising the cap increases progressivity. Comparing two people above the cap, raising the cap decreases progressivity. As most people are below the cap, it can be argued that the former case is more significant.