## FINAL EXAM, PUBLIC FINANCE (4054) NAME:

You must show correct work for credit. Put boxes around your answers for algebraic and numerical problems.

1. Tax incidence. Suppose that there is a perfectly competitive market with aggregate demand that can be represented by the marginal benefit function  $MB(q) = 8 - \frac{1}{100}q$ , where q is the quantity consumed. Suppose further that, for each unit of the good that they sell, suppliers must pay a tax of \$2 to the government. In each of the following three scenarios, find the following: (a)  $P^*$ , the price that buyers would pay and sellers would receive *in the absence of the tax* 

- (b)  $P_d^*$ , the price that buyers pay when the good is taxed
- (c)  $P_s^*$ , the price that sellers receive, *net of the tax* ( $P_s = P_d \tau$ )
- (d) DWL, the deadweight loss associated with the tax

**1-1.** Aggregate supply can be represented by the marginal cost function  $MC(q) = 2 + \frac{1}{100}q$ .

1-2. Aggregate supply is perfectly elastic, due to a constant marginal cost function, MC(q) = 4.

**1-3.** Aggregate supply is perfectly inelastic at a quantity of  $\overline{q_s} = 300$ .

2. Tax incidence – algebra. Suppose that there is a perfectly competitive market in which marginal benefit is given by  $MB(x) = \alpha - \beta x$ , and marginal cost is given by  $MC(x) = \gamma + \delta x$ . Suppose that suppliers must pay a tax of  $\tau$  for every unit they sell. Assume that all of the model's parameters are positive, i.e. that  $\alpha > \gamma > 0$ ,  $\beta > 0$ ,  $\delta > 0$ , and  $\tau > 0$ .

**2-1.** Find the equilibrium quantity  $x^*$ , in terms of the parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ , and  $\tau$ .

**2-2.** Find  $P^o$ , the price paid by buyers and received by sellers *in the absence of the tax,* in terms of the parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ , and  $\tau$ . Also, find  $P_d^*$  (the equilibrium price paid by buyers, including the tax) and  $P_s^*$  (the equilibrium price received by sellers, *net of the tax*), in terms of the same parameters.

**2-3.** Find  $\Delta P_d \equiv P_d^* - P^o$  and  $\Delta P_s \equiv P_s^* - P^o$  in terms of the parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ , and  $\tau$ . Briefly comment on the way in which the parameter values determine the direction of the inequality  $|\Delta P_d| \leq |\Delta P_s|$ , and what this says about which side of the market will bear the greater burden of the tax. **3. Taxation and labor supply.** Ariel has a job selling newspapers, which pays \$10 per hour. Her utility function can be represented by  $U(C, L) = CL^2$ , where C is consumption per day and L is leisure per day. She can neither save nor borrow, and she has no other income, so her consumption each day must be equal to her income from work. Let H be the number of hours that Ariel chooses to work; since there are 24 hours in each day, it must be true that L + H = 24.

**3-1.** First, consider the case in which Ariel is not charged any tax at all. Find her optimal choices of work hours per day  $(H^*)$ , leisure per day  $(L^*)$ , consumption per day  $(C^*)$ , and utility per day  $U(C^*, L^*)$ .

**3-2.** Second, consider the case in which Ariel is taxed \$5 for each hour of work, or 50% of her wage income. Again, find her optimum choices of work hours per day ( $H^*$ ), leisure per day ( $L^*$ ), consumption per day ( $C^*$ ), and utility per day  $U(C^*, L^*)$ . Also, find the tax revenue that the government collects from her every day ( $R^*$ ).

**3-3.** Third, suppose that, instead of taxing Ariel \$5 per work hour, the government simply charges her a lump sum tax of \$45 per day, whether she works or not. Again, find her optimum choices of work hours per day ( $H^*$ ), leisure per day ( $L^*$ ), consumption per day ( $C^*$ ), and utility per day  $U(C^*, L^*)$ . How does this lump sum tax compare with the tax on wages (in 3-2) in terms of revenue and utility?

## 4. Taxing and spending – algebra.

**4-1.** In problem 2-1, you found an expression for the equilibrium quantity  $x^*$  in terms of the parameters of a linear supply and demand model with a per unit tax, i.e.  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ , and  $\tau$ . Using this as your starting point, find an expression for  $DWL(\tau)$ , the deadweight loss associated with the tax, and  $R(\tau)$ , the revenue derived from the tax, in terms of the same parameters.

**4-2.** Suppose that  $(1 + \lambda)$  represents the shadow value of an additional dollar of tax revenue, and that your goal is to maximize the objective function  $\Omega(\tau) = (1 + \lambda)R(\tau) - [R(\tau) + DWL(\tau)]$ , i.e. the gain in revenue value from taxing this market, net of the lost surplus. Find the per unit tax  $\tau^o$  that accomplishes this, as a function of the parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ ,  $\tau$ , and  $\lambda$ .

**4-3.** Suppose that there is some public good which (because of the free rider problem) can only be provided effectively by the government, using tax revenue. Aggregate marginal benefit for the public good is given by MB(y) = A - By, and the marginal cost of providing the public good is given by  $MC(y) = \Gamma + \Delta y$ , where y is the quantity of the public good provided. Supposing that  $\Lambda$  represents the shadow cost of raising revenue for public expenditure, find an expression (in terms of A, B,  $\Gamma$ ,  $\Delta$ , and  $\Lambda$ ) for the public good quantity  $y^o$  that maximizes the objective function  $\Psi = TB(y) - \Lambda E(y)$  (where TB and E are total benefit and expenditure, respectively), by setting  $MB(y) = \Lambda MC(y)$ .

5. Application of simple linear model to labor taxation. This is effectively an extension of problems 4-1 and 4-2, but instead of a generic goods market with parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$ , suppose that we are dealing with a series of markets for individuals' labor. Each person *i* has the same marginal cost of labor function  $MC(h_i) = \gamma + \delta h_i$ , where  $h_i$  is the number of hours he or she works. The demand for each person's labor is perfectly elastic, but wages are heterogeneous; thus, demand for person *i*'s labor can be represented by the function  $MB(h_i) = w_i$ . Assume that the government can apply a different per hour labor tax  $\tau_i$  to each person *i*.

**5-1.** Modify your answer to 4-2 above to give an expression for  $\tau_i^o$ , the efficiency-maximizing tax on each individual, in terms of  $w_i$ ,  $\delta$ ,  $\gamma$  and  $\lambda$ . Then, use this to derive an expression for  $t^o \equiv (\tau_i^o h_i^*) \div (w_i h_i^*)$ , i.e. total tax as a share of total wage income for each person. If all workers must pay a tax of  $\tau_i^o$  per unit of labor, under what circumstances (in terms of the parameters) is this a *proportional* tax? (Assume that no one has income from other sources.) Under what circumstances is it a progressive tax?

**5-2.** In the context of taxation, what is the difference between equity and efficiency? How might you adapt the tax structure from 5-1 to one that takes equity into account as well as efficiency? Try to be as precise and clear as possible in your answer.

6. Taxes on income from saving. Consider a simple two period model in which Stan earns an income of Y in the first period, and no income in the second period (retirement). Thus, first period consumption is given by  $c_1 = Y - S$ , where S is saving, and second period consumption is given by  $c_2 = (1 + nr)S$ , where r is the interest rate that Stan is paid for his savings, and  $n \equiv 1 - t$  is the share of his interest income that remains after taxes. r > 0,  $n \in (0, 1)$ , and the tax rate  $t \in (0, 1)$ .

**6-1.** Use algebra to create a budget constraint in the form  $p_1c_1 + p_2c_2 = I$ , where  $p_1$  and  $p_2$  are in terms of exogenous parameters, and *I* does not contain  $c_1$  or  $c_2$ . Using your expressions for  $p_1$  and  $p_2$ , comment on whether  $c_1$  or  $c_2$  is relatively 'cheaper'.

**6-2.** Suppose that Stan's utility is given by the function  $U(c_1, c_2) = \alpha \ln c_1 + \beta \ln c_2$ . Find his optimal choices of  $c_1^*$ ,  $c_2^*$ , and  $S^*$ , in terms of the parameters  $\alpha$ ,  $\beta$ , *Y*, *n*, and *r*.

**6-3.** Suppose that the tax rate t increases (causing n to decrease). Comment on the income effects of this and the substitution effects of this, in terms of whether each of them causes Stan to increase or decrease his savings. Use your answers from 6-2 to determine whether the income effect or the substitution effect is dominant given that particular utility function.

**7.** Describe four provisions of the 2010 Patient Protection and Affordable Care Act (PPACA), including one source of new funding used to offset its new expenditures.

**8.** Why is the individual mandate considered the 'linchpin' of the PPACA? What particular problems would be associated with implementing the remaining provisions of the law *without* the individual mandate?

**9.** How does the federal estate tax in the US currently work (in 2011)? Who pays it? Which estates do and do not have tax liability?

**10.** Provide a brief argument in favor of the estate tax. Feel free to make note of rebuttals to your specific points, if applicable.

**11.** Provide a brief argument against the estate tax. Feel free to make note of rebuttals to your specific points, if applicable.

12. What is the earned income tax credit, and how does it work?

**13.** Given that the government has full information about firms' costs and revenues, what would be the effect of a 75% tax on the *economic profits* of all firms?

**14.** Explain one way in which corporate income earned overseas receives a tax advantage relative to corporate income earned domestically.

**15.** Name two reasons why it's typically cheaper to buy health insurance from a (large firm) employer than as an individual.

**16.** Explain one way (independent of the marginal rate) in which capital gains income receives a tax advantage relative to labor income.

**17.** Explain a possible advantage of taxing real estate property according to the estimated value of the land alone, as opposed to taxing it according to the estimated value of the entire property including its improvements (e.g. structures).