Problem Set 4, due Thursday, October 15

1. Profit maximization with one input. Suppose that a (perfectly competitive) firm has the production function $y = f(x) = 12x^{1/3}$, where x and y are the firm's input and output quantities, respectively. Let p be the price of the firm's output, and let w be the price of the firm's input. Assume that the firm has no other costs.

a) Write the $\pi(x)$ function (profit as a function of the input quantity x) in the most explicit possible form, and use this function to solve for the profit-maximizing input quantity, x^*, in terms of p and w.

b) Write the $\pi(y)$ function (profit as a function of the output quantity y) in the most explicit possible form, and use this function for the profit-maximizing output quantity, y^*, in terms of p and w.

c) Use each of your calculations above to find expressions for R (revenue), C (cost), and π (profit) when the firm is choosing its input and output in a profit-maximizing manner. Again, each of these depend only on p and w. Verify that you can the same answers with both methods.

2. Profit maximization with two inputs. Suppose that a (perfectly competitive) firm has the production function $y = f(x_1, x_2) = {x_1}^{1/5}{x_2}^{1/5}$, where y is its output quantity, and x_1 and x_2 are the quantities it uses of inputs 1 and 2, respectively. Let $p = 10{,}000$ be the output price, and let $w_1 = 1$ and $w_2 = 4$ be the input prices.

a) Your goal is to find the profit-maximizing values of the input and output quantities: x_1^*, x_2^*, and y^*. There are two main ways to do this; explain how each of these methods works.

b) You have explained two methods; now, implement them. Find the numerical values of x_1^*, x_2^*, and y^*, and verify that you get the same answers with both methods.